COMAP'S
EXPLORING MATH MODELING IN AND OUTSIDE OF THE CLASSROOM WEBINAR SERIES

Webinar #6

"What? No Answer Key?": Building Creativity & Confidence in Student Math Modelers

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@COMAPMATH

What Modeling Looks Like at Our Schools

David	Greta	Cynthia
Senior Seminar in Statistical Research (Two-semester stats and modeling course) Math Modeling club (focused on COMAP and M3 Competitions)	Standalone modeling course (morphed into AI / Computational Modeling) which participates in HiMCM and M3. Modeling embedded in upper grades (focus on applications)	Extracurricular Club--meets once a week, competes 2-3 x each year Club Presents to other classes in late Spring Principles and Skills incorporated into classroom

"Word Problem" …..vs. Modeling

- The Population of "Yourtown" is 20,000 people
- 35% of its citizens recycle their plastic water bottles.
- Each person uses 9 water bottles per week.
- How many bottle will be recycled each week in Yourtown?
- Your answer…..?

…vs. Modeling Problem:

- How much material is recycled in Anytown?
- How successful is the recycling program?

- Your answer….?
- Your QUESTIONS….?

Building a model: The Celebrity Divorce Formula

1. Defining variables—input

2. Meaningful units, domain & range

3. Expressing relationships

   (Direct, inverse, joint variation)

4. Defining variables-- output

   (Evaluation/rank , Decision, Plan)

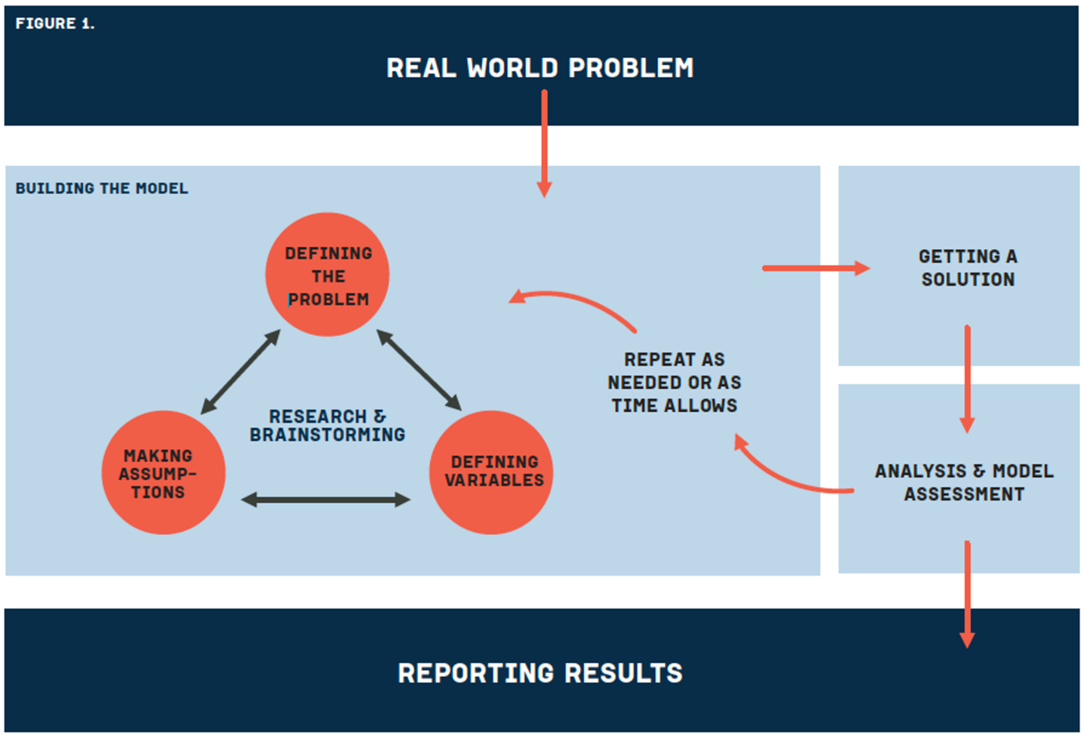
$$2\left(\frac{30 - \mathbf{S}^2}{\mathbf{P} + 5}\right)\left(\frac{10\mathbf{A}_b + 10\mathbf{A}_g}{(\mathbf{A}_b - \mathbf{A}_g)^2 + 100}\right)\sqrt{\frac{75(10 + \mathbf{G}_b)}{(\mathbf{G}_b + \mathbf{G}_g)\,(10 + \mathbf{G}_g)}}\left(\frac{\mathbf{D}}{\mathbf{D} + 2}\right)^{2T} = \mathbf{B}_{liss}$$

P= The couple's combined number of previous marriages
$\mathbf{A}_b$= His age in years
$\mathbf{A}_g$= Her age in years (biological, not cosmetic)
$\mathbf{G}_b$= In millions, the number of hits when Googling his name
$\mathbf{G}_g$= In millions, the number of hits when Googling her name
S= Of her first five Google hits, the number showing her in clothing (or lack thereof) designed to elicit libidinous thoughts
D= Number of months they knew each other before getting married (enter a fraction if necessary)
T= Years of marriage. To find the likelihood of their marriage surviving 1 year, enter 1; for the likelihood of it lasting 5 years, enter 5, etc.

$\mathbf{B}_{liss}$ is the percentage chance that this couple's marriage will last for the number of years you chose.

Source: "From Tinseltown to Splitsville: Just Do the Math"
New York Times (September 19, 2006

The Modeling Process



Problem Resources

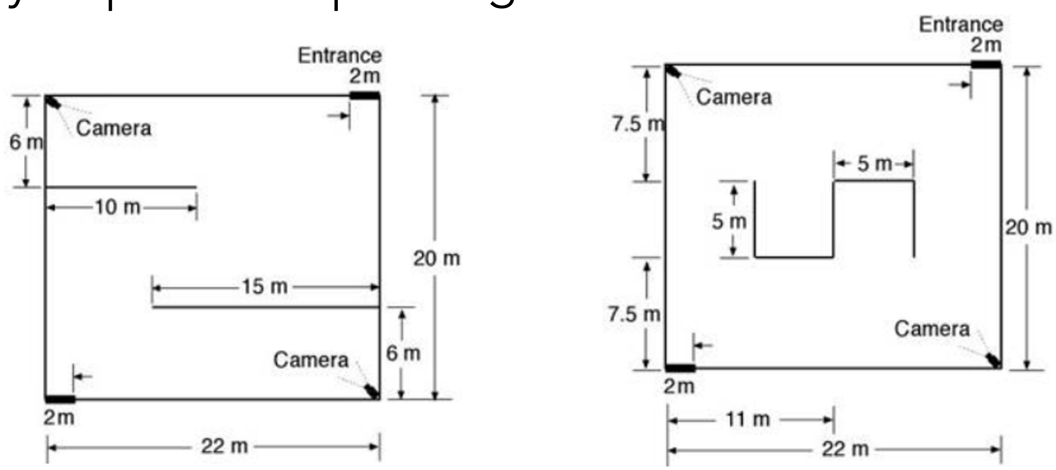
MODELING COMPETITIONS

- CoMAP www.comap.com
  - High School Mathematical Contest in Modeling (HiMCM)
    - First contest in 1999
  - Mathematical Contest in Modeling (MCM)
  - Interdisciplinary Contest in Modeling (ICM)

- SIAM https://m3challenge.siam.org/
  - Mathworks Math Modeling Challenge (M3 Challenge)
    - First contest in 2006

Example: The Art Gallery Problem

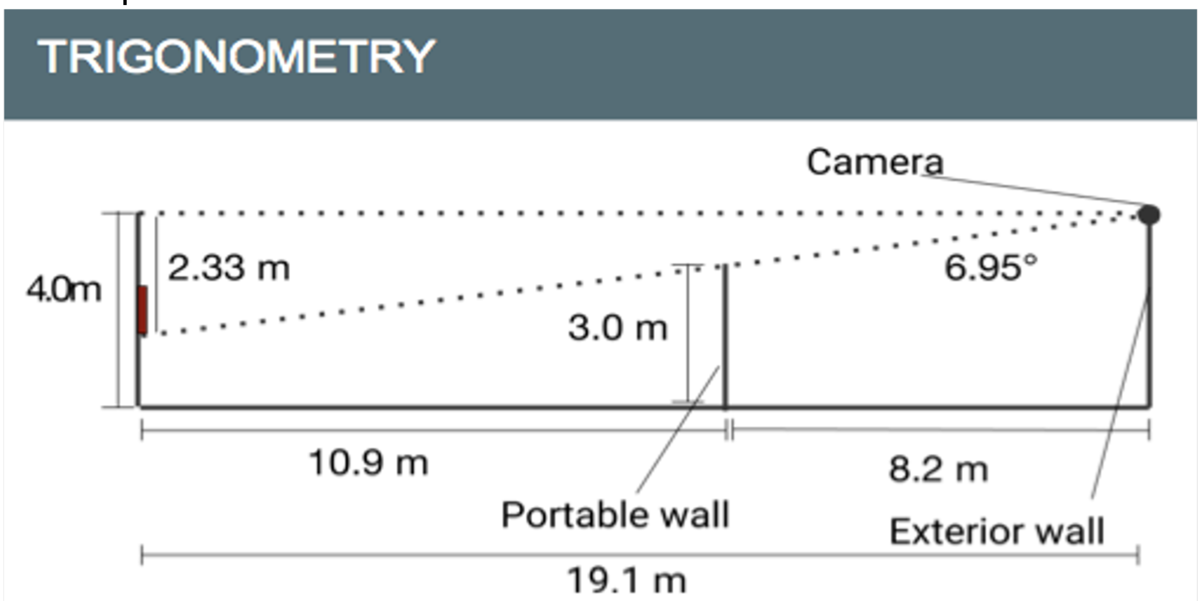
An art gallery needs to hold 50 paintings. Security cameras rotate at opposite ends of the room. The outer walls and portable walls have given measurements, and the layout should be aesthetically pleasing. Where should you place the paintings in order to maximize security?



HiMCM Problem A, 2004

Differentiation – Art Gallery #1

- One group made such detailed trigonometric diagrams that you could put them in a precalculus textbook!



Courtesy Lauren Shareshian

Differentiation - Art Gallery #2: Calculus

Another group used calculus concepts to quantify security

Let R be a given room, and C be a given timing for the cameras

- $V_{R,C}(t)$ is visible painting count at time t given R and C
- Security is therefore

$$S_R = \lim_{t_{max} \to \infty} \max_{C} \frac{1}{t_{max}} \int_0^{t_{max}} V_{R,C}(t)dt$$

- This is the average number of paintings seen per second as time goes to infinity

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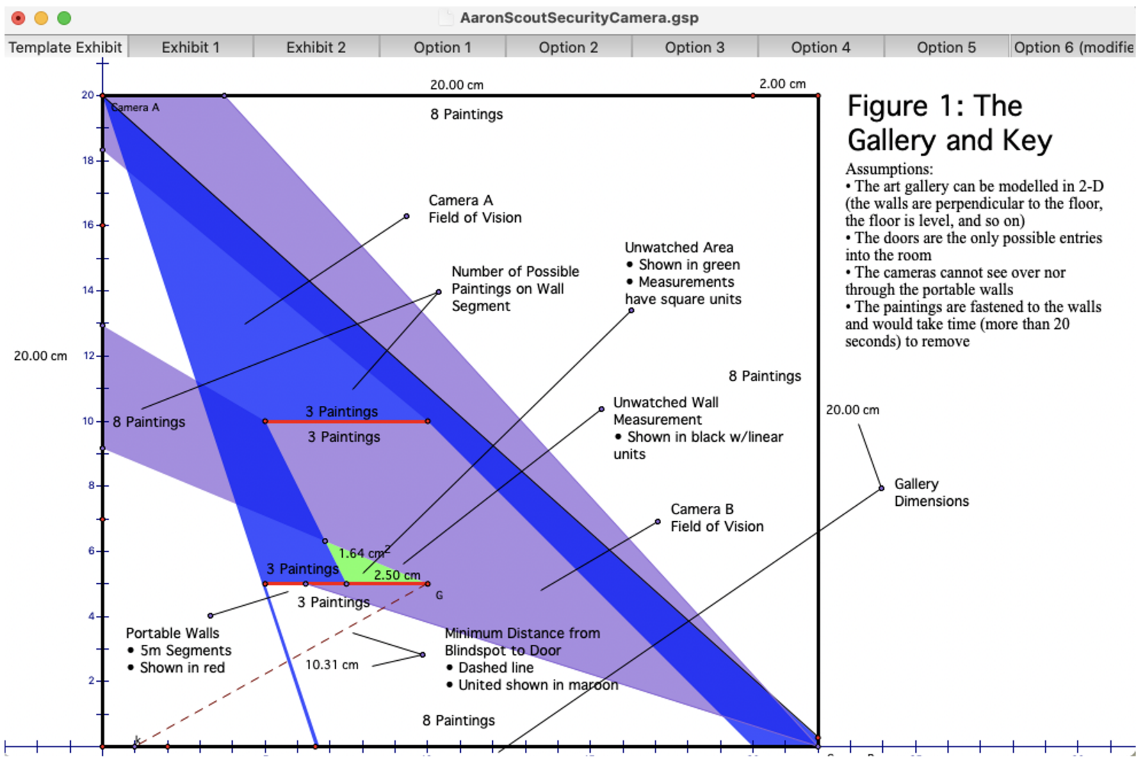
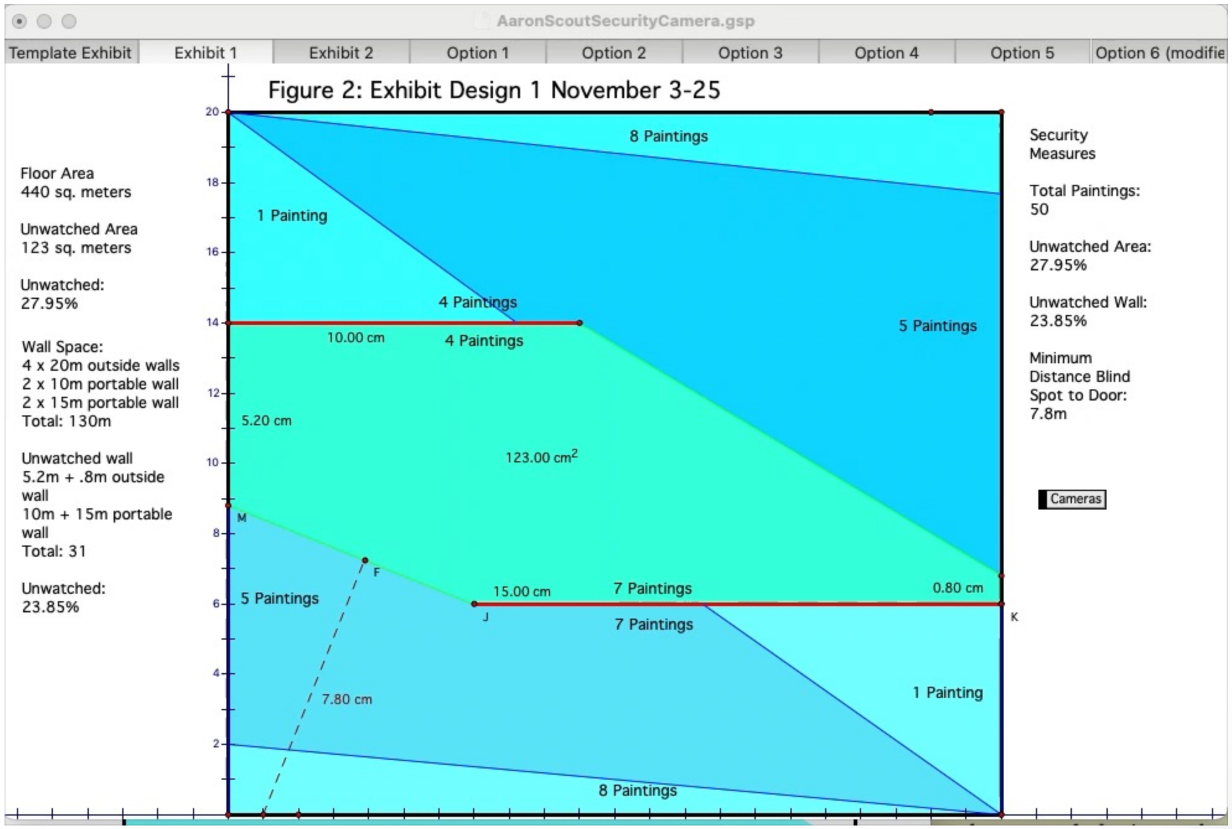
Courtesy Lauren Shareshian

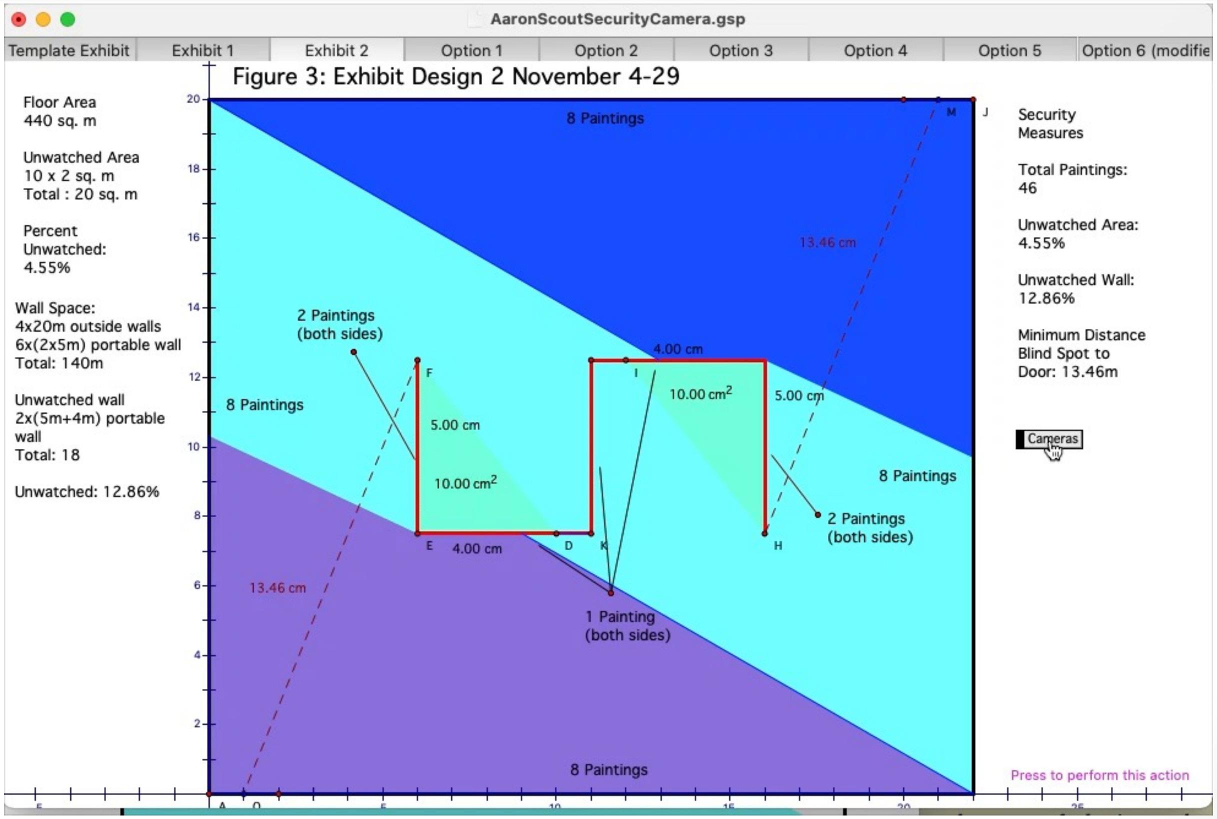
Differentiation – Art Gallery #3: Dynamic Geometry Software



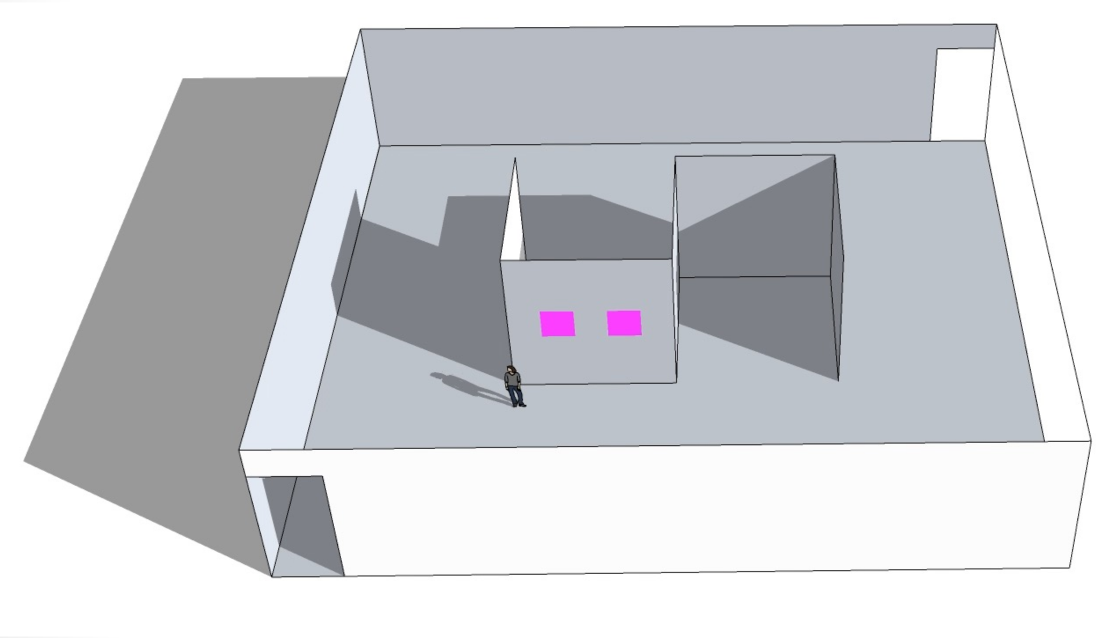
Differentiation – Art Gallery #3: Dynamic Geometry Software



Differentiation – Art Gallery #3: Dynamic Geometry Software



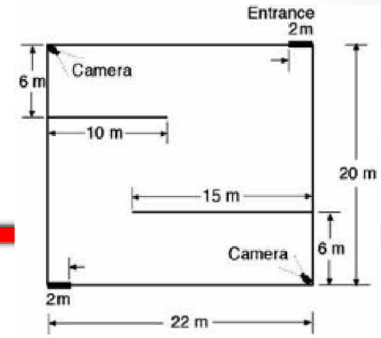
Differentiation – Art Gallery #4: Modeling in 3D



Additional notable approach: one team modeled the problem by developing a playable video game. The player takes the part of the art thief and moves through the gallery trying to escape detection. The metric is the player's final score – a lower score = greater security

Modified Problem

Figure 1: Exhibit Configuration
November 5 – 27, 2011



Your task is to address the following:

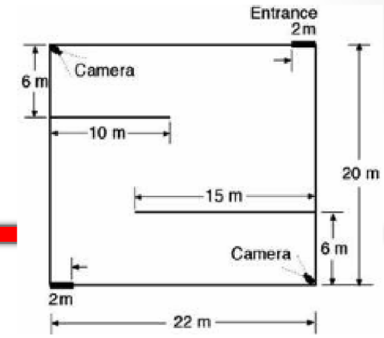
- *What does it mean to be "secure?"* You will determine a way to measure the security of the exhibit for different wall configurations. The impact of the motion of the camera on the security of the exhibit should be factored into your metric in some meaningful way. **Your metric should make use of trigonometric formulas.**

- *How secure were the previous exhibits?* Use your metric from I to determine the security of the previous exhibit.

- *What is the most secure layout for the upcoming exhibit?* Determine an optimum portable wall configuration for the watercolor exhibit based on **<u>your</u>** measure of security.

Modified Problem

Figure 1: Exhibit Configuration
November 5 – 27, 2011

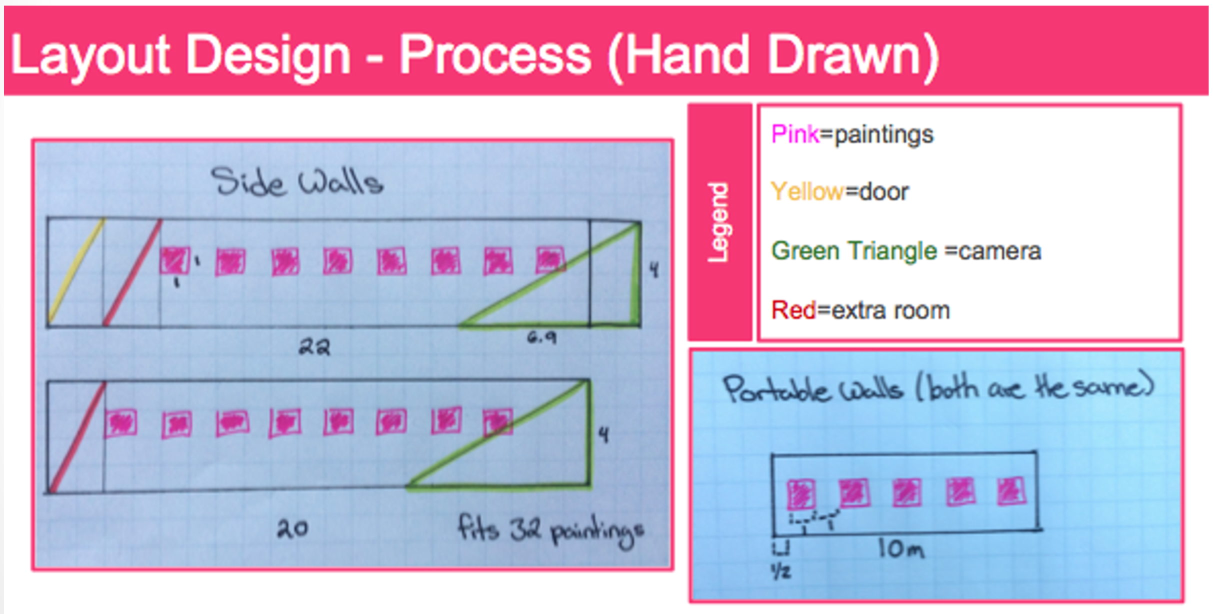


Additional required elements:

- For both exhibits, include diagrams showing the cameras' position at 4 different times: 0 seconds, 10 seconds, and two other times between 0 and 10 seconds.
- Calculate the security of each exhibit using your metric.
- Write a summary of your project that introduces the problem, briefly explains your security metric, and includes your final results. The summary should NOT be a "play-by-play" of your project.
- Your group will have 10 - 15 minutes to present your project to your classmates.
- Creativity is encouraged!

Modified Problem – Art Gallery #5

- Low-tech but effective (cardboard box and pencil and paper)



Courtesy Lauren Shareshian

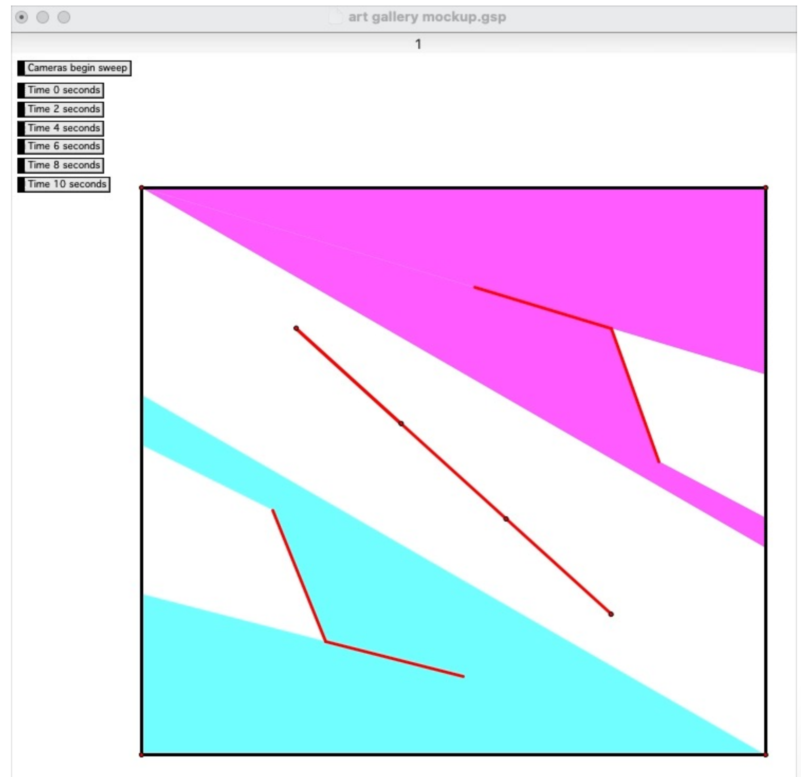
Differentiation – Art Gallery #6

To find amount of visible wall space:

- Right triangle trigonometry
- Law of Sines / Law of Cosines

To find % of visible area:

- Area formulas using trigonometry



Adapting Your Curriculum for Modeling

Protocol Shared at

Wisconsin Math Council Annual Meeting, May 2018

Thanks to:

Paige Jones, Hudson High School

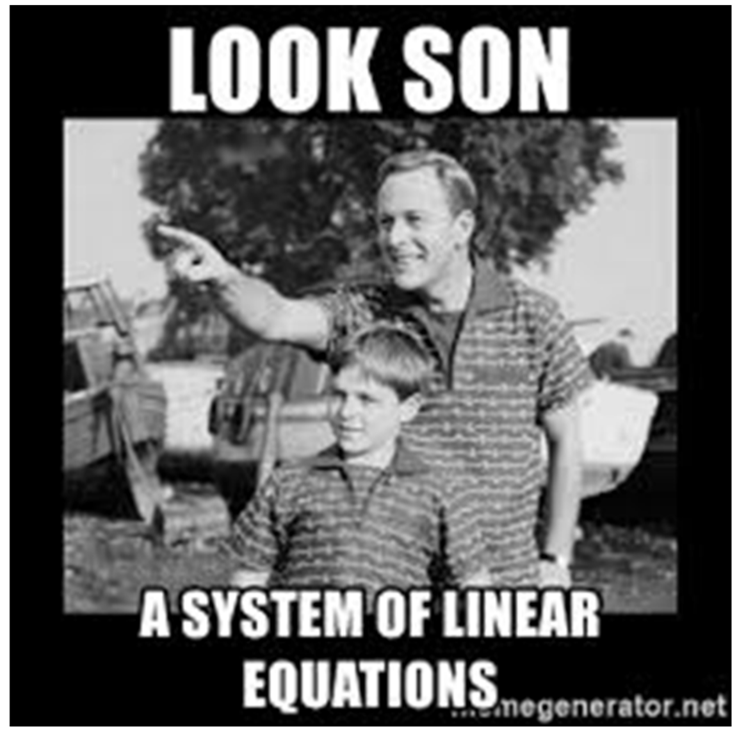
Samantha Falkner, Eau Claire Memorial High School

Naked Math Problem

Solve the system.

$15x + 6.5y = 84$

$2x = y$



Adding LABELS, Context

$15x + 6.5y = 84$

$2x = y$

Sarah and her family decide to go and see the new "Avengers" movie in theaters this weekend. It costs $15 per adult and $6.50 per child. It cost her family $84 for the tickets. If there were twice as many children as adults in Sarah's family, how many adults and how many children attended the movie?

Adding freedom

Your summer family reunion committee has $200 budgeted for the movies.

How many people can see the new “Avengers” film, and what part of the week should you schedule this outing?

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Student Voice and Choice

- Modeling empowers students to make real decisions
- Modeling gives each student the opportunity to participate according to their strengths
- Modeling engages all students by offering problems that are of genuine interest and / or concern to students
- Modeling validates student efforts by ensuring that there is an authentic audience ("client")
- Modeling provides context for math and its applications

Common Core State Standards

"Modeling is defined as both a conceptual category for high school mathematics and a mathematical practice and is an important avenue for motivating students to study mathematics, for building their understanding of mathematics, and for preparing them for future success."

CCSS: Math Practice 4

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

Mathematically proficient students are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions.

They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Tips for Teachers

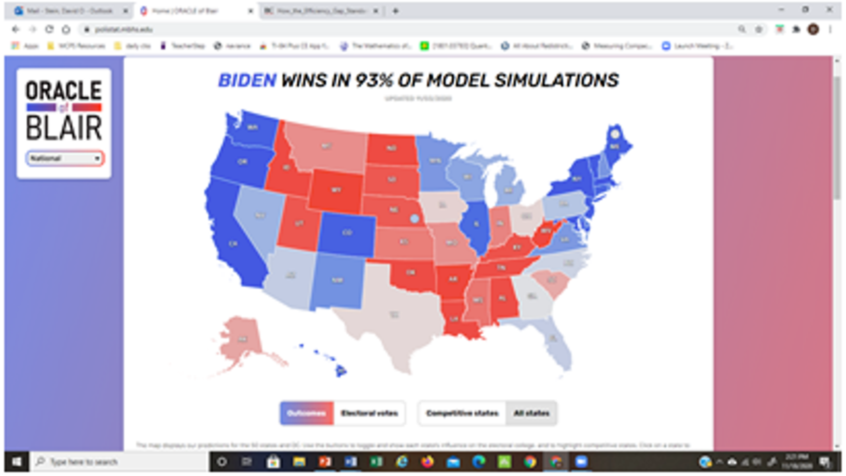
- All work is group work.
- No one can do everything but everybody can do something
- Find projects where they can put their models to the test
- Go where their interests lie
- Teacher as an editor instead of a grader.
- Make communication a big part of the assignment
- Data Viz – pictures worth a thousand words

Tips for Students

- Thinking through assumptions can be the most important part
- Make sensitivity analysis central to the thinking
- Training sets AND testing sets.
- Simple models can be just as interesting as complex models
- Uncertainty is just as important as point estimate. Maybe more.
- Think about effect sizes – coefficients and confidence intervals.
- Making a model in one field might help your thinking a great deal in another.

ORACLE OF BLAIR

Oracle of Blair



Resources

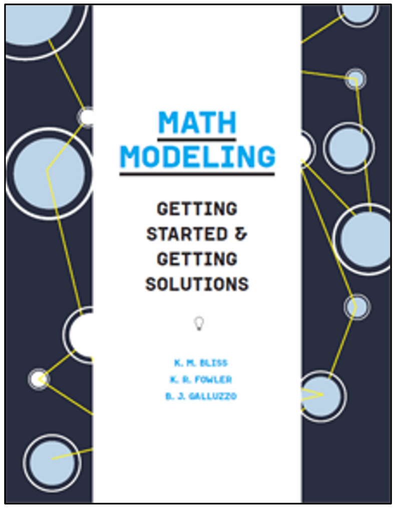
MATH Models

http://www.mathmodels.org/

Math Modeling Hub

https://qubeshub.org/community/groups/mmhub

Resources


MATH MODELING
GETTING STARTED & GETTING SOLUTIONS
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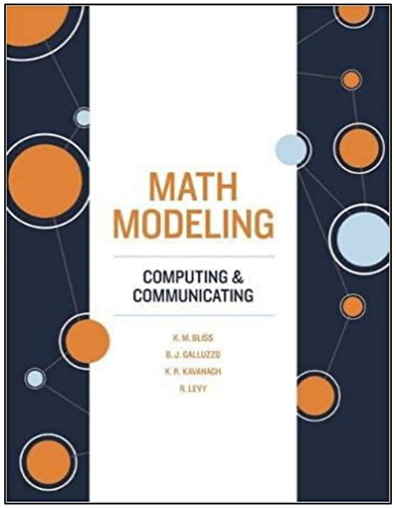
GAIMME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Questions, Comments, Discussion