The 2020 HiMCM

Dr. Kathleen Snook, HiMCM Director

Congratulations to our 2020 Outstanding team winners and to all teams participating in our twenty-third International High School Mathematical Contest in Modeling (HiMCM). We are excited to again join with the National Council of Teachers of Mathematics (NCTM) to designate two of our Outstanding teams as NCTM Award winners. The HiMCM continues to be an amazing and rewarding experience for students, advisors, schools, and judges across the globe. A total of 779 teams, with up to 4 students each, representing 302 schools and 20 countries /regions, competed this year.

Outstanding Teams

- 10549 Basis International School Guangzhou, Guangdong, China
- 10550 Nanjing Foreign Language School Xianlin Campus, Jiangsu, China
- 10656 Massachusetts Academy of Math and Science at WPI, MA, USA (NCTM Winner, Problem A)
- 10701 Shenzhen Foreign Languages School, Guangdong, China
- 10839 The McCallie School, TN, USA
- 10876 Shanghai Pinghe School, Shanghai, China
- 10997 Hwa Chong Institution, Singapore (NCTM Winner, Problem B)
- 11135 Buchholz High School, FL, USA

In this year of challenges with virtual and hybrid learning environments, we are truly impressed with the amazing teamwork shown by participating students. We commend all students and advisors for the creativity and ingenuity of their mathematical efforts. It appears that teams truly enjoy continuing to develop mathematical models to address our HiMCM problems.

The 2020 Contest

Our 2020 participating teams submitted some truly impressive papers. We see the vision of our founders in the *unique* and *creative mathematical* solutions to complex open-ended real-world problems. As in the past, students chose from two problems. This year's problems challenged teams to assist other high school students in finding a summer job in Problem A: *The Best Summer Job*, or to determine the priority and method of funding of various endangered plant species' projects in Problem B: *Funding Biodiversity Conservation*.

Overview

In 2020, COMAP celebrated their 40th birthday and the HiMCM its 23rd contest. As more high schools engage their students in mathematical modeling, we hope participation in COMAP's modeling contests will follow. Starting with 115 students in the first year of the HiMCM, over the course of 23 contests we have had 38,585 students apply their mathematical knowledge and skills as they modeled challenging problems in the HiMCM.

The mathematical modeling ability of participating students continues to be evident in the problem solutions and professional submissions we receive. We acknowledge and credit advisors and teachers for their work with these students. As teachers and students engage in mathematical modeling at a higher level, we are happy and excited to assist your efforts. Let us know how COMAP might support your modeling activities.

The 2020 contest had 824 registered teams resulting in 779 total submissions (94.5%), a submission rate three-percent higher than 2019. Of the 779 submissions, 501 completed Problem A: *The Best Summer Job*, and 278 completed Problem B: *Funding Biodiversity Conservation*. **Table 1** shows the judging results of the 2020 HiMCM. We encourage all registered teams to submit a final solution paper in order to experience the learning impact and satisfaction of fully completing this challenging contest.

In total, 2878 students competed in the 2020 HiMCM. A wide range of schools competed, including teams from Australia, Brazil, Canada, Chile, China, Finland, Germany, Hong Kong (SAR), India, Indonesia, Italy, Malaysia, New Zealand, the Philippines, Singapore, South Korea, Turkey, the United Kingdom, the United States of America, and Wales, UK. This was an increase from 18 to 20 represented countries/regions. The 207 teams from the United States represented 25 states. Submissions included 572 foreign teams. China represented about 89% of the foreign participants.

Problem	Outstanding	%	Finalist	%	Meritorious	%	Honorable Mention	%	Successful Participant	%	Unsuccessful/ Disqualified	%	Total
A	5	1%	38	8%	60	12%	157	31%	235	47%	3 / 3	1%	501
B	3	1%	18	6%	34	12%	86	31%	132	47%	0 / 5	2%	278
Total	8	1%	56	7%	94	12%	243	31%	367	47%	3 / 8	1%	779

Table 1: 2020 HiMCM Judging Results

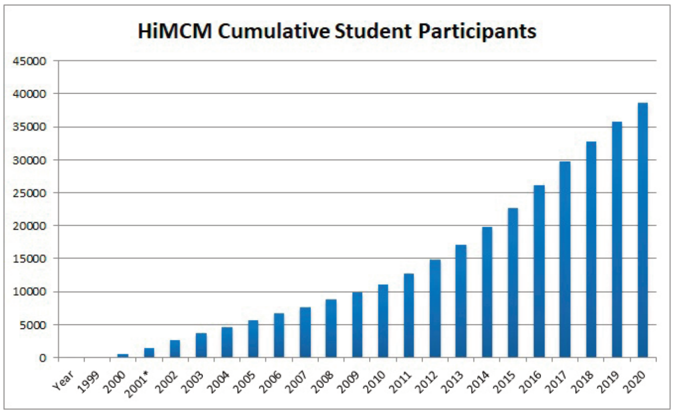


Figure 1: Total HiMCM Student Participants 1999 – 2020

(Note: there were two HiMCMs in 2001 when contest moved from spring to fall.)

Of the 2878 student participants this year, 809 (28%) self-identified as female, 1549 (54%) self-identified as male, and 520 (18%) did not specify gender. Since the start of HiMCM in 1999, the 38,585 total participants are quite diverse. We hope that all competing students enjoy their contest experience and continue to pursue further Science, Technology, Engineering, and Mathematics (STEM) education. We welcome all levels of high school students to the HiMCM. The only prerequisites for success in the contest are high school mathematics skills and concepts. **Figure 1** shows the growth of the HiMCM with the total number of students participating since the contest's start in 1999.

Rule Changes

Our recent rule changes, with regard to the contest window and scheduling working time for team members, provided our teams flexible work hours resulting in high quality paper submissions for both problems. Our change in 2020 to limit submission documents to 25 pages resulted in well-written and well-organized solution papers. Teams must choose what information, modeling, and graphical support is necessary to present their full analysis of the problem within 25 pages.

One rule that has never changed is that students are only to use the members of their own team along with inanimate (non-living) sources. Students may not use any chat rooms, electronic communication, or social media sources. Each year we have some teams that do not understand this rule. To be clear, contacting an expert in a field or an author of one on the referenced sources is a violation of this rule. Gathering data from persons outside of your team through the use of an interview or a survey or a questionnaire is a violation of this rule. Using solutions shared electronically by other teams or by organizations is a violation of this rule. Again, only the team members may contribute to the solution through their own knowledge and work, and by using inanimate resources (e.g. research articles, web sites, textbooks, journals, publications). Additionally, COMAP will *never* require that you purchase additional materials or information to be successful in the HiMCM. The materials and information provided by COMAP, along with your own team's knowledge, skills, and perhaps a bit of research using allowed references, is all that you need for success.

We are in our second year of using Twitter and Weibo—
Follow us @COMAPMath on Twitter
or COMAPCHINAOFFICIAL on Weibo for contest guidance and up to date contest information.

Judging

All contest submissions are electronic. This allows us to form a high quality judging pool from academia and industry. In December 2020, the pandemic resulted in using only remote preliminary judges. Remote judges were located in the US states of Alabama, California, Connecticut, Georgia, Maryland, Massachusetts, Nevada, New York, North Dakota, Ohio, Pennsylvania, South Carolina, Texas, Virginia, and Washington. Each paper is read by two preliminary judges. We thank these judges for their careful review of our HiMCM submissions.

All judging is blind with respect to any identifying information about the participants or their schools. Preliminary judges ranked papers as Finalist, Meritorious, Honorable Mention, and Successful Participant. Judges sent all papers ranked as "Finalist" to a virtual Final Judging. This year, 65 papers were forwarded to Final Judging for a panel of thirteen judges to consider. As these 65 papers were the best submissions from the preliminary round, at final judging the judges chose the "*best of the best*" as Outstanding papers. Eight papers earned the Outstanding award.

The final judges commended the preliminary judges for their efforts in selecting the high quality Finalist papers. We feel that the structure of preliminary and final judging provides a good process for identifying the top-level papers.

The Future and The New MidMCM

For the past 22 years, the HiMCM has sought to provide all high school students the opportunity to compete and achieve success in applying mathematics. Our efforts have focused on meeting this important goal. Mathematical modeling continues to grow within the high school curricula across the globe, and we recognize that middle school students are now modeling too! COMAP is very excited to announce that in November 2021, a middle school contest option – the International MidMCM – will be available during HiMCM. The MidMCM will allow middle school aged students the opportunity to demonstrate their mathematics and modeling abilities in this new contest. Please visit

www.MidMCM.com

for more details about this initiative.

More than just learning skills and operations, mathematics is both an art and a science. Through mathematical modeling, students learn to think critically, communicate effectively, and be confident, competent problem solvers. Success is not only about the procedural technique used, but the conceptual understanding in discovering the role of assumptions and model development in driving those techniques to a valid solution and conclusion.

Advisors and students often ask what level of mathematics is required, and what special programming or coding skills are needed for the contest. To be clear, all of our HiMCM problems are accessible using high school level mathematics alone, and no programming or coding skills are required or necessary. Our new MidMCM problems will require only middle school level mathematics. As in all of our contests, each of our problems is accessible on multiple levels. Students should apply the mathematics they understand and are able to explain in their solution analysis.

The ability to recognize problems, formulate a mathematical model, use technology, and to communicate and reflect on one's work are important skills to develop. Applying the mathematical principles and concepts that one learns is key to individual and societal future success.

The MidMCM will join the HiMCM in providing a vehicle for using mathematics to build models that allow students to represent, and to understand, real world behavior in a quantitative way. Both contests will enable student teams to look for patterns and think logically about mathematics and its role as a language in our daily lives. Students gain confidence by tackling ill-defined problems and working as part of a team to generate a solution. We are excited that in our contests, applying mathematics is a team sport.

Advisors need only be motivators and facilitators to encourage students to be creative and imaginative. COMAP encourages all middle and high school mathematics faculty to get involved, encourage your students to be problem solvers, make mathematics relevant, and open the doors to future success. We want to partner with teachers as we continually strive to improve the contest, and make it accessible and impactful to all students. Any school can enter and each school can enter as many teams as that school desires. MidMCM and HiMCM have no restrictions on the number of total schools or the number of total teams.

—Awards—

Outstanding

Finalist

Meritorious

Honorable Mention

Successful Participant

After final judging, MidMCM and HiMCM papers will receive a designation in one of the categories above. Depending upon the quality of the papers, the top 20–25% of submitted papers receive a designation of Meritorious or above, with approximately the top 1% designated as Outstanding.

2021 Contest Dates

Mark your calendars for the next HiMCM, and the first MidMCM, to be held November 3-16, 2021. Registration for the 2021 MidMCM and HiMCM will open in September. As described previously, recent contest rule changes allow teams to have the flexibility to schedule their work efforts during the contest window. At the team members' convenience, teams download and choose their problem, complete their modeling solution, and electronically submit their solution document by the deadline on November 16th. Again in 2021, one Outstanding team for each problem will receive the NCTM award. Teams can learn more about COMAP's contests and registration via the Internet at http://www.comap.com.

MathModels.org

Powered by COMAP content, Mathmodels.org is a wonderful resource for students and teachers to make math modeling a year round activity. Teachers and students may use the materials found on this site to enrich their classes and help prepare students for mathematical modeling competitions. We encourage you to visit www.mathmodels.org.

The International Mathematical Modeling Challenge, IM²C

The International Mathematical Modeling Challenge (IM²C), held each spring, continues to grow. The purpose of the IM²C is to promote the teaching of mathematical modeling and applications at all educational levels for all students internationally. It is based on the firm belief that students and teachers need to experience the power of mathematics to help better understand, analyze and solve real world problems outside of mathematics itself – and to do so in realistic contexts. Each country/region administers the contest for its own students and then sends its top two teams to the international final judging. An Expert Panel of final judges determines winners and also selects teams to present their solutions at an international award ceremony. In 2020, selected teams presented at local, regional, or virtual conferences due to COVID-19. IM²C hopes to return to an international award ceremony in 2021. To learn more about the IM²C visit www.immchallenge.org for rules and country/region contacts.

COMAP invites teams from the United States that successfully competed in the HiMCM contest at the designation of Meritorious or above (Meritorious, Finalist, and Outstanding) to compete in IM²C US Regional Round. Registration is free! From these participants, our US IM²C judges select the two top teams to represent the USA in the IM²C international round. See US Rules for IM²C at https://immchallenge.org/Pages/Rules/USA/USA-Rules.html.

2020 Problem Discussions and Judges' Commentary

The following paragraph describes what our preliminary and final judges look for in identifying and judging competitive HiMCM papers:

> Regardless of the problem chosen, competitive papers include a comprehensive summary, address all requirements through developing and applying a mathematical model, and write a clear letter or memo if required. Better papers do all of the above in an articulate, well-supported, well-organized, and well-presented manner. The best papers combine complete mathematical and logical analysis, and explain their work in an organized presentation beyond simply addressing the requirements. These best papers are easy to read, flow logically, and they include sections that address assumptions with justifications, the modeling process(es), results of modeling and analysis, strengths and weaknesses, sensitivity, conclusions, and references.

Our judges have asked that I continue to stress that all of our HiMCM problems are written to be accessible by students at any level of high school mathematics. Some teams attempt to use advanced concepts and tools found on the Internet that they do not explain clearly or use appropriately. Judges recognize this, and these papers do not do well. We are not looking for papers that use the most advanced mathematics. We have found that the best papers develop a mathematical model that incorporates high school level mathematical concepts and tools that the teams are able to fully explain, use appropriately, and analyze subsequent results. The most important aspects of solutions are the model development, and the clear use and analysis of the model toward addressing the requirements of the problem.

The specific problem discussions below provide comments on how teams addressed the requirements of each problem. Following this section we provide the judges' comments about the solutions and presentations by breaking down the various parts of a submission and providing exemplars. To view the complete problem statements visit

www.mathmodels.org

or

www.himcmcontest.com.

Problem Discussions

Problem A: The Best Summer Job

In Problem A, we asked teams to think about opportunities high school students have for a summer job. Realizing that there are many different types of jobs with many different factors to consider, we first asked teams to identify (list and describe) the various factors high school students should consider in their job search. For example, jobs may have varying required hours, or hourly rates, or types of work (e.g. physical or sedentary), or required skills (e.g. analytical or organizational). Teams used their lists of factors to develop a model or algorithm (or set of models/algorithms) for a high school student to use to evaluate summer job options based on a student's own situation and preferences. Once teams had their model, they created a group of at least ten fictional and diverse students, each of whom had fictional desires, preferences, and skills. Teams tested their model using their fictional persons to choose a job for each. Teams also analyzed the results of the application of their model. Finally, teams described a method to present and use their model via a webpage or an app or a newspaper article.

Most teams began by doing some research on summer jobs for high school students. From this research they developed a list of factors to consider. As the problem notes, and the teams discovered, factors were qualitative and quantitative, constant and variable, and deterministic and probabilistic. In order to be competitive, teams had to list the basic factors provided in the problem statement and describe them well. Better papers included and described a more comprehensive list of factors. Factors included some of the following: hourly rates, hours per week, activity level, required skills, virtual or in-person, COVID-19 risks, transportation requirements, and costs (for meals and transportation). These papers included a broader scope of considerations (e.g. paid jobs versus internships). The best papers listed and described many factors and then discriminated among all factors through analysis to come up with those to include in their model. Through some method of factor analysis, these teams may have combined one or more similar factors into a single more general factor. For example, a salary index based on hourly rate, transportation and meal costs, and possible overtime pay.

Once teams identified summer job factors, they had to develop a reasonable mathematical model (or models) to input a person's preferences within the chosen factors and to output a job or type of job. At this point, to be competitive, teams had to recognize an unwritten requirement to develop a list of potential jobs, or at least types or categories of jobs, for high school students. Teams also had to develop the "values" of each of their chosen factors for each job/category in their list. Teams had to recognized that the problem required some matching algorithm to best match the differing personal preferences and skills to the various job opportunity characteristics. Better papers were clearer in describing their model development processes to include support for decisions about how to incorporate the various factors and preferences into their model, which factors to include, how to include them, and if/how to weight the factors. Many papers included some sort of scoring between the personal preferences and the job offerings in terms of the chosen factors. The best papers addressed other nuances within the factors and preferences. These papers addressed variations occurring in different jobs and so developed a model to match persons to a group or type of job versus a specific job.

The students seemed to have some fun in developing the ten fictional persons to test their models. Competitive teams used the factors in their model and developed fictional persons who presented variation in those factors. Many times, teams included a chart or table of their persons and factors. Teams then used their model to find the best job for each fictional person. Better papers similarly chose their ten persons, but added a bit more about each person (e.g. qualitative aspects in addition to quantitative values). These richer test cases allowed these teams to better match a person to a job opportunity. The best papers had rich descriptions of their test cases and if necessary, to ensure they comprehensively covered the variations in their factors, they included additional fictional persons. These teams fully supported each decision of both factors and persons. By doing this, these teams were able to more confidently match each fictional person to a type of job.

Teams described how they would present their model for other high school students to understand and use. Teams used webpages, apps, written instructions, and articles. Competitive papers presented and described their model clearly. Better and best papers enhanced their presentation using graphics or flowcharts and their descriptions were easy to follow and understand.

Problem B: Funding Biodiversity Conservation

Problem B provided teams with background on the plight of thousands of species of plants and animals facing extinction. Although actions to save all these species are often available, there is limited funding to do so. The Florida Rare Plant Conservation Endowment (FRPCE) supports research, protection, and conservation of rare and imperiled plant species in Florida.

COMAP and HiMCM thank the researchers from the Arizona State University and from the State of Florida shown in **Table 2** for assistance in providing data and expertise in the development of this real-world problem.

Funding Biodiversity Conservation

Olivia Davis	Arizona State University
Gwen Diacona	Arizona State University
Dr. Leah Gerber	Arizona State University
Dr. Vivian Negrón-Ortiz	U.S. Fish and Wildlife Service

Table 2: Problem B Development Team

Problem B required teams to determine how to efficiently invest in biodiversity conservation activities for endangered and threatened species that take place over long time frames, and whose expected costs change over that time. Teams considered 48 imperiled plant species.

We asked teams to develop a model to advise the FRPCE toward "long-term and reliable" funding. In an initial analysis of the problem, teams identified and discussed objectives the FRPCE should consider in their efforts and budget decisions. They also addressed some general characteristics of imperiled plant species to use in their decision model. Finally, teams developed a model or algorithm (or set of models/algorithms) for the FRPCE to use to determine a fundraising schedule. The schedule was to show the funds required with a timeline for fundraising, while minimizing funds to be raised and obtaining necessary resources to implement and manage the 48 projects.

Competitive teams recognized that while the FRPCE objectives included saving the most species, they also included prioritizing the species. Teams recognized that necessary funds had to be available for projects to begin or continue. While competitive teams may have realized that that not all projects could be done at the same time and that all projects were not equal in funds required or in impact, it was in the better team's papers that these realities became an integral part of the model. These papers addressed variations in the start times of projects to minimize the funds necessary in any particular year, as well as variations in priority of effort. The best papers developed a model that comprehensively looked at targeted species and their characteristics to specifically meet the objectives of the FRPCE. These papers had a model that was clearly developed and presented, and could be used to determine a viable fundraising schedule.

Once teams had a model, they applied their model to the actual 48 plant species in the data and recommended a priority order of funding and a timeline for these particular projects. Competitive papers used and applied their models to the species' data provided and suggested fundraising targets and a timeline. These papers were organized around funding priority projects first and funding as many projects as possible per year. The better papers broadened their approach to be flexible in both scope and funding during any particular year, but to use their model to suggest a fundraising plan. For example, some teams attempted to schedule projects so as to level out required annual funds as much as possible and have level fundraising targets each year. Other teams conducted fundraising every five years, using principal and interest to support projects until the next fundraiser. Better teams also realized that although each project's data were given over a period of years (the life of the project), that projects could start in any year and the budget and fundraising timeline could extend beyond 25 years into the future. The best papers developed a comprehensive model that informed their decision about the priority order of projects and the requirements and time-line of fundraising. They presented clear analysis of their model development, its use with the given data, and their recommendations.

Finally, teams wrote a one-page non-technical memo to the FRPCE explaining their results and making recommendations. Most teams were able to write an informative letter to the FRPCE. Non-technical letters should be clear in their explanations and not include too many mathematical details. For example, you definitely want to let the FRPCE know that you developed a priority list for the 48 species and you might list your recommendation of the top several species to save, but you do not need to include your mathematical formula for doing so in the memo.

Judge's Discussion

While the problem discussions above provide comments on the solutions to this year's problems, in the following paragraphs we examine the sections of a submission and provide comments about the solutions and the presentation of the solutions from our judges' point of view. At the end of the article, we have included excerpts from our Outstanding papers as exemplars. Mathmodels.org members can view all the unabridged versions of the Outstanding papers online.

Overall

Participants must ensure their papers follow the contest rules posted on the contest website. Papers that are coherent, organized, clear, and well written provide a great impression to the judges. The logic and mathematics of these papers are easy to follow. Teams should present their entire submission in 25 pages or less, using at least 12-point font. These 25 pages should include your introduction/executive summary, your solution that addresses all requirements, a resource list, and any appendices. While students may want to include some background research on the problem topic, this information should be brief. It is not the number of pages, but the ability to complete all contest requirements and communicate the solution in a concise and articulate fashion that will merit recognition. Students should use spelling and grammar checkers before submitting a paper. Foreign papers should insure that all symbols in tables and graphs are in English. Student and school names should not appear on solution papers.

Papers considered for Finalist and Outstanding start with a clear summary that describes the problem. These teams then preview their paper with an organized Table of Contents. They present assumptions with justifications, explain the development of their model and its solutions, apply their model, and support the results mathematically. These best papers communicate all of the above clearly, do a sensitivity analysis, address strengths and limitations, and finally, close by stating overall conclusions.

Executive Summary and Introduction

Judges are best able to analyze a paper when students restate the problem in their own words and clearly preview the focus and organization of their paper. Teams should write the executive summary after they finish their solution as they summarize the entire contents of the paper. Teams should consider a three to five paragraph approach for their summary: a restatement of the problem and questions in their own words, a short description of their method and solution to the problem (in words and not in mathematical expressions), and the conclusions providing the numerical answers in context. The executive summary should entice the reader, in our case the judge, to read the paper. Although written last, ensure you spend time on this important part of your submission. Your executive summary is the first page the judges will read and provides the first impression of your paper. **Example 1** presents a good summary for Problem B: *Funding Biodiversity Conservation*.

The comments above for summaries also apply to this year's job model presentation in Problem A and species' memo in Problem B. The presentation or memo might briefly describe your model or process, but should do so in a non-technical manner, as the reader may not have a mathematical background. The job model presentation was to attract high school students to understand and use your model. A nice cover for the job model presentation for Problem A: *The Best Summer Job* is shown as **Example 2** with a presentation as **Example 3**. The purpose of the species' memo was to focus on results and to make recommendations to the FRPCE. A good memo for Problem B: *Funding Biodiversity Conservation* is shown as **Example 4**.

Following the summary, the paper should provide some brief background, restate the problem, and perhaps provide a preview of the solution. Teams should not simply repeat what was in the summary. In **Example 5**, a team provided an introduction, problem restatement, and a preview of their modeling process to include a schematic of the structure of their work for Problem A: *The Best Summer Job.* **Example 6** illustrates similar introductory sections and a model preview for Problem B: *Funding Biodiversity Conservation* followed by Assumptions with Justifications. Using a flow chart or diagram is an excellent way to present your overall model or algorithm in a succinct manner. Many times a picture is worth a thousand words! Both **Examples 5** and **6** include diagrams of their planned modeling processes.

Assumptions with Justification

Modeling assumptions should include only those that come to bear on the solution (to assist with simplifying your model). Good and relevant assumptions are difficult to identify and articulate. Long lists of assumptions that do not play directly in the context of model development or its solution are not considered relevant and deter from a paper's quality. Assumptions that oversimplify the problem too much do not allow for a full solution. You should include a short justification to show the assumption is reasonable and necessary. The end of **Example 6** shows a list of assumptions with justifications this team deemed required for their particular modeling process in Problem B: *Funding Biodiversity Conservation*.

Mathematical Model

The development of the mathematical model is the most important part of your HiMCM submission. Papers should explain the development of the mathematical model(s) and/or algorithm(s), and define all variables. Teams that merely present a model without explaining or showing the development of that model do not generally do well. Although in the course of your work you may have developed several models, presenting multiple models without identifying the most appropriate model to answer the questions is detrimental to your paper's success. Clearly present the development and results of your chosen or final model. If you include a long list of variables in a table early in your paper, consider reminding the reader of the variable definitions as you present various parts of your model. This is very helpful to the judges so they can follow the logic of your model. Judges do value creativity and thinking "outside of the box" in your modeling process. There is always more than one appropriate solution method to our HiMCM problems. So, be creative.

To get started on their modeling in Problem B: *Funding Biodiversity Conservation*, teams needed to determine the objectives of the FRPCE and identify characteristics of the plant species. **Example 7** shows how one team addressed these requirements in preparation for model development.

Perhaps the most important step of the modeling process is the last one: <u>explicitly present your final model in its full form</u>. And, if needed, remind the reader of the variables you are using in this model. Too many times, judges remark that after working through a team's development of their model, they cannot find a clear presentation of the final full model that the team will use in the follow-on requirements.

There are many ways to model and analyze the HiMCM problems. This year we saw many appropriate, as well as creative, models to address both problems. We include a model development example for each problem. **Examples 8** presents model development for Problem A: *The Best Summer Job*. **Examples 9** presents model development for Problem B: *Funding Biodiversity Conservation*.

Strengths and Limitations

Teams should address strengths and limitations in evaluating their model and solution, and include model extensions or sensitivity analysis of the solution. Teams should validate their models, even if by numerical example or intuition. **Example 10** provides strengths, weaknesses, and sensitivity analysis for Problem A: *The Best Summer Job*. Another sensitivity analysis for Problem A: *The Best Summer Job* is shown in **Example 11.**

Conclusion

A clear conclusion and answers to the specific scenario questions are key components to an Outstanding paper. Attention to detail and proofreading the paper prior to final submission are vital as the judges look for excellence in your submission. See a nice discussion of recommendations and conclusion from Problem A: *The Best Summer Job* at **Example 12**.

Citations and References

Citations are very important within the paper, as well as either a reference list or bibliography page at the end. Teams that use existing models should cite their source within the paper at the point they present the model and also include a reference citation in the back of the paper. This is also true for all graphs and tables taken from the literature. Use "in line" documentation with footnotes or endnotes to give proper credit to outside sources. All data, figures, graphs, and tables that come from outside sources require documentation at the point in the paper where they appear. Lack of documentation will result in a lower designation. We have noticed an increase in the use of Wikipedia. Teams need to realize that although useful, information from Wikipedia might not be accurate. Teams should recognize and acknowledge this fact and look for primary resources.

The quality of HiMCM submissions continues to improve each year. Our high school students are truly amazing! We enjoy reading all of the papers submitted for review and are truly impressed by the work of the student teams. We encourage teams to review the comments and guidance provided in this Consortium article and to visit mathmodels.org in preparation for next year's contest. Also, follow us

@COMAPMath on Twitter

or

COMAPCHINAOFFICIAL on Weibo

for information about all COMAP contests.

List of Examples:

1. Summary (Problem B, Team 10997, Hwa Chong Institution, Singapore)
2. Cover Sheet (Problem A, Team 10701, Shenzhen Foreign Language School, Guangdong, China)
3. Presentation (Problem A, Team 10549, Basis International School Guangzhou, Guangdong, China)
4. Memo (Problem B, Team 10997, Hwa Chong Institution, Singapore)
5. Introduction (Problem A, Team 10701, Shenzhen Foreign Language School, Guangdong, China)
6. Introduction (Problem B, Team 10876, Shanghai Pinghe School, Shanghai, China)
7. Model Development (Problem B, Team 10876, Shanghai Pinghe School, Shanghai, China)
8. Model Development (Problem A, Team 10549, Basis International School Guangzhou, Guangdong, China)
9. Model Development (Problem B, Team 10839, The McCallie School, TN, USA)
10. Sensitivity, Strengths and Limitations (Problem A, Team 11135, Buchholz High School, FL, USA)
11. Sensitivity Analysis (Problem A, Team 10550, Nanjing Foreign Language School Xianlin Campus, Jiangsu, China)
12. Conclusion (Problem A, Team 10656, Massachusetts Academy of Math and Sciences at WPI, MA, USA)

Summary

As global biodiversity continues to face an unprecedented crisis imposed by habitat loss and invasive species, endangered plant species conservation is crucial for restoring environmental equilibrium. Conservation managers nowadays are facing difficult decisions due to limited resources and numerous projects awaiting funding. In this problem, we are tasked with coming up with a priority order of funding for the given 48 plant conservation projects under the Florida Rare Plant Conservation Endowment (FRPCE) Board.

Before developing our model, we first identify the relevant objectives that the Board would want to meet in deciding the fundraising schedule. Firstly, they would want to **maximize expected net benefit**, which is represented by a weighted score incorporating benefit, cost, taxonomic uniqueness and feasibility of the selected projects. Since we expect the Endowment to have a relatively constant annual revenue, they would also prefer schedules with **minimal fluctuations in yearly spending**. To meet these objectives, we identify the common characteristics of imperiled plant species and interpret the factors involved in their conservation.

In our model development, we establish a feasibility decay function to model the increasing risk of extinction if conservation actions are not taken or delayed. This is accomplished by logistic population growth with Allee effect. Next, to determine which projects to prioritize, we assign a **Priority Index** to each project in each year, which takes into account the project's benefit, taxonomic uniqueness, feasibility of success, total cost, and duration. Priority Index is determined using the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and Entropy Weight Method. Subsequently, we employ Dynamic Programming with Greedy Algorithm to obtain an initial schedule that maximizes the sum of Priority Index of selected projects. The schedule specifies the projects that the Board should prioritize to fund in each year, and displays the required yearly expenditure.

Genetic Algorithm is then used to optimize our initial schedule. We use a comprehensive **total score** as our objective function, which incorporates both the expected net benefit as well as the standard deviation of our proposed annual funding schedule. The algorithm performs crossover, mutation, and tournament selection to reach a schedule with the best total score. After iterating 300 generations, we obtained our optimal schedule. For a schedule with a maximum funding cap of $500,000, the use of Genetic Algorithm can reduce standard deviation in yearly spending from 109443 to 39215.

To suggest to the Board an optimal fundraising schedule that can minimize funds raised and achieve long-term and reliable funding, we choose **$500,000** as the funding cap. In case they want to complete all the projects, we also provide them with the schedule with a funding cap **$2,000,000**. Lastly, we present our **priority order of funding** in a color-coded table which specifies the starting year of each project.

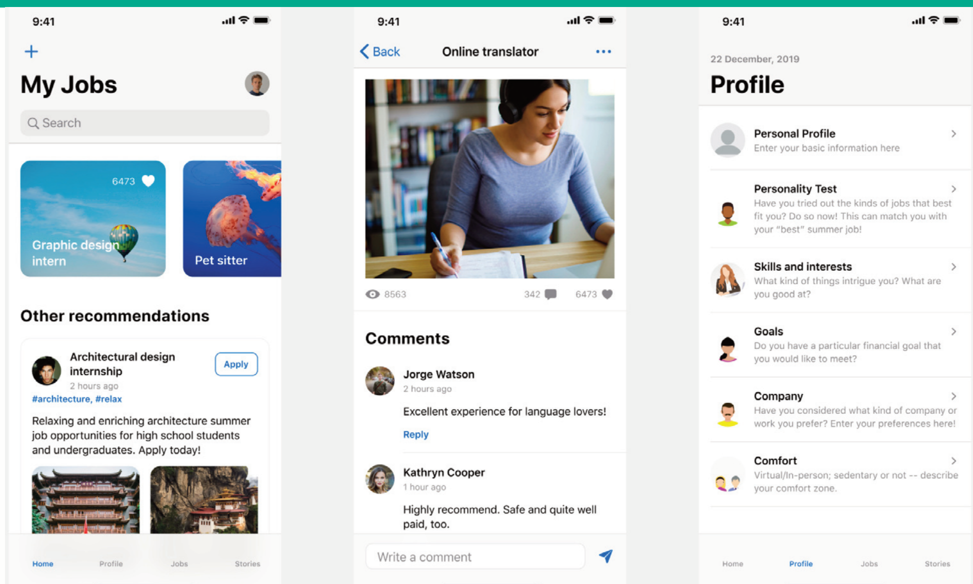
Keywords: Conservation of Endangered Plant Species, Logistic Model with Allee Effect, TOPSIS Enhanced by Entropy Weight Method, Dynamic Programming with Greedy Solution, Genetic Algorithm

Example 1: Problem B Summary
Team 10997, Hwa Chong Institution, Singapore

Example 2: Problem A Cover Sheet
Team 10701, Shenzhen Foreign Language School, Guangdong, China

NOVEMBER 17,2020

GET A JOB!



HOW TO FIND THE BEST SUMMER JOB?

Have you ever felt frustrated from finding the best summerjob for you? Well, you are not alone. Now due to the COVID-19outbreak, it becomes even more difficult for high school students to find suitable summerjobs.

Thankfully,you don't have to stress about that now! Our research constructs a straightforward model for high school students to use. Our model allows you to simply enter your personal information and goals for the job, and it will quickly return the best summerjob options for you! Our model incorporates jobs' income, comfort, skill level, company size, and risks, as well as your personal situations and preferences, to make the best summerjob recommendation. Furthermore, our app will offer you an opportunity to view other students' comments on the summerjobs, allowing you to further decide on which job would be the best for you personally.

What are you waiting for? Come and find yourself a job today!

SUMMER JOBS RECOMMENDER

BEAUTIFUL LAYOUT, STRAIGHTFORWARD MECHANISM, AND EFFECTIVE RECOMMENDATIONS

According to our research, teen summer job has declined in the US. This model will provide personalized job recommendations for high schoolers.

Example 3: Problem A Presentation
Team 10549, Basis International School Guangzhou, Guangdong, China

8 Memo

Date: 17/11/2020

To: The Florida Rare Plant Conservation Endowment (FRPCE) Board

From: HiMCM Team # 10997

Subject: Recommendation of funding schedule for 48 imperiled plant species conservation

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As a group of students passionate about mathematical modeling, we recently learnt about the Board's consistent efforts in funding imperiled plant species conservation projects in Florida. We are also aware of the difficulty you are facing in prioritizing the various recovery projects. Given the data set of 48 species' conservation projects, we utilized mathematical models and computer algorithms to generate funding schedules that can maximize conservation benefits, and at the same time, minimize fluctuations in yearly expenditures.

In quantifying an aggregate expected net benefit score, we take into account the feasibility of success, benefit, taxonomic uniqueness, and total costs of each project. A good funding plan should achieve a high project success rate, save species with the most conservation value. We also recognize that the Endowment would not like to spend too much or too little money on a specific year, wasting or straining its yearly budget. Hence, our proposed plan is optimized to have relatively stable yearly funding.

Based on our algorithm, we suggest $500,000 as the annual funding cap since 43 out of 48 projects can be completed and the schedule is the most cost-effective. Meanwhile, we present $2,000,000 as the minimum yearly funding that could finish funding all projects in a reasonable time span. The priority order of funding we obtained are listed in Figure 14. Under the specific annual funding column, the number corresponding to a project indicates the start year of the project. For instance, 1-FloweringPlants-135 should be funded starting from Year 2 when the Board adopts the $500,000 funding cap. When the value is NC, this conservation project is not chosen, usually because the yearly cost greatly exceeds the annual budget or the expected net benefit is too low.

You can understand the rationale behind the schedule easily. In our model, we first used a Priority Index to decide which projects to be funded first. Projects with greater benefit and uniqueness are prioritized since they yield more conservation value. We also prioritize shorter projects and those which require less funding to accommodate for more projects in the future. Feasibility of a project was modeled to be decreasing logistically with time, which means the feasibility of a project decreases the fastest near the median value. To achieve a high overall evaluation score, we prioritize these projects since an extremely high/low feasibility does not change appreciably with time. Based on our results, projects like 1-Flowering-Plants-502/481 are done during the first year as they have low total cost and short time span, fair benefit and uniqueness as well as intermediate feasibility values. Projects with very high feasibility like 514/179 are funded later since they are less urgent. Our model was tested to have the maximum expected net benefit and minimum fluctuations in yearly expenditures.

We hope that by selecting and implementing the appropriate funding schedule from our results table, your organization will be able to bring maximum welfare to Florida's rare plants and their habitat. An optimal funding schedule is crucial to conservation efforts in Florida given pressing extinction threats imposed by climate change, land use change and urban development. Species conservation still has a long way to go but we hope that our efforts can contribute to your decision-making process to save endangered species.

Example 4: Problem B Memo
Team 10997, Hwa Chong Institution, Singapore

1 Introduction

Summer job is a type of job that students do in summer vacation, allowing students to gain experience in their interested field and also earn money. Students are capable of further improving their academic performance, learning about important employment skills, and being immersed in real-world working circumstance ahead of time. What is more, summer jobs enable students to find a meaningful way to relax instead of simply staying at home. To be more detailed, students can refresh their minds and boost their energy through the experience of working for the jobs they are interested in during summer vacation. This is probably because they will gain a sense of personal satisfaction, as they may do something that makes contributions to society[1]

Also, for many young people who struggle with economic burden, they are likely to face the prospect of a difficult transition into work or college. Applying for a job in the current labor market enables them to benefit from being able to perceive why educational attainment is important, besides the income they gain. To fulfill the increasing demand for summer jobs, there are a considerable amount of summer jobs available for students who have an interest in finding a summer job to reach certain goals.

2 Problem Restatement and Analysis

The way people evaluate whether a summer job is suitable for them or not normally depends on lots of factors, including inner factors, such as personal income and fields they are interested in, and external factors, such as the wage and working environment. However, due to the great number of summer jobs existing and the various kinds of factors which people should take into account, people tend to be frustrated when they are choosing what kind of summer job they should participate in. There are indeed some summer job evaluation forms provided by several institutions. However, these forms cannot provide a clear report or evaluation to people, since they only ask people some questions without forming a suggestion about their choices. Thus, there is clearly a need for a comprehensive model to help people evaluate the value of various summer jobs based on their personal interests and the benefit they will gain from this experience.

To solve this problem, we will make a new evaluation system and rank each summer jobs to help students to find the “best” jobs. In our model, based on the preference and personal situation of different students, subjective evaluation methods need to be used.

We are also asked to develop at least ten fictional persons. The purpose of doing this is to ensure our evaluation system to be global and well fit to students around the world. To make these ten persons typical representatives of students around the world, an online survey will be conducted to collect personal information from teenagers. In the end, we need to develop an app to illustrate how we will promote our model to the general public.

3 Our Work

The process of our work first contains introducing the topic and giving a further exploration into the problem. This helps us to better understand the purpose of solving the problem. Next, we need to purpose the assumptions and justification. After dividing the indexes we need to consider, we will use TOPSIS and AHP to evaluate and analyze our data. In order to make our model results reliable and readable, we develop ten fictional persons. After making the sensitivity analysis, we make a thorough consideration of our users’ experiences, both recognizing our app’s advantages and disadvantages. Finally, based on our model results and app’s design, we state our strengths and weaknesses.

Example 5: Problem A Introduction
Team 10701, Shenzhen Foreign Language School, Guangdong, China

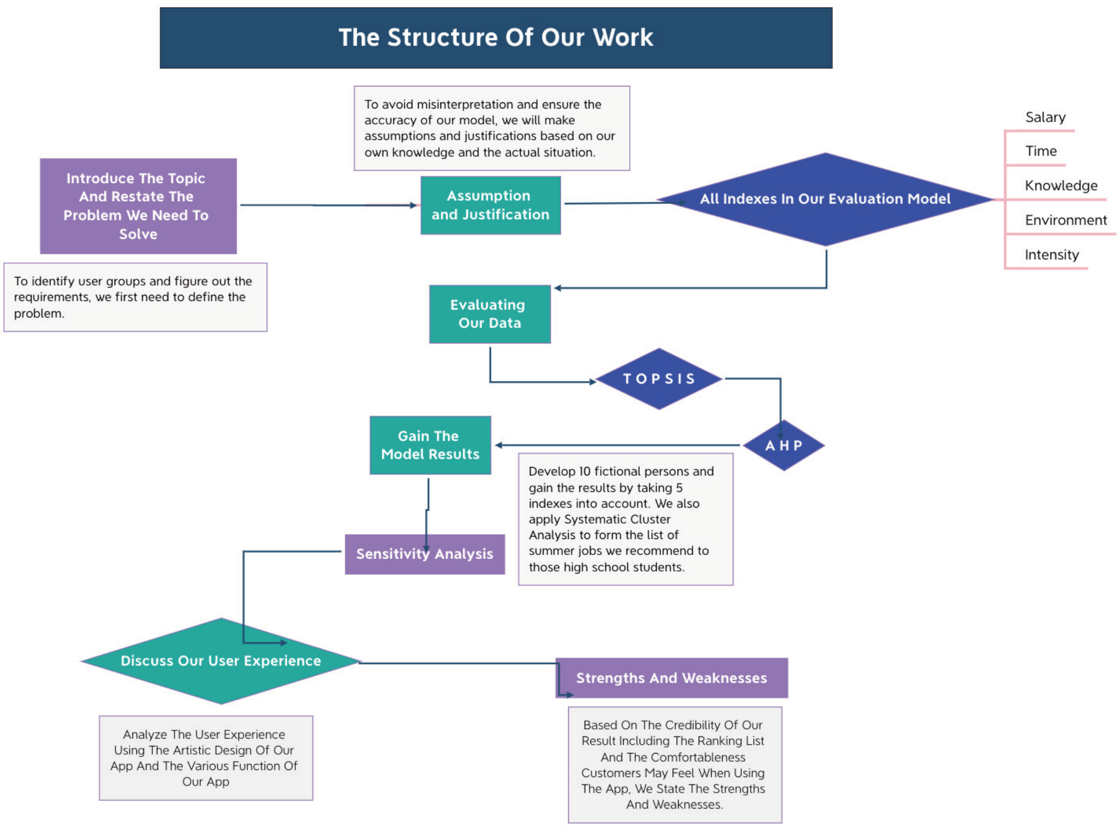


Figure 3.1: Work Structure

1. Introduction

1.1. Background

Whereas human society is developing at an unprecedented pace, plants and animals is left to face the disastrous environmental consequences. Legislations, as well as fund donations and public campaigns, help fighting plant and animal extinctions. Signed into law in 1973 by President Richard Nixon, the Endangered Species Act (ESA) is credited with saving America's national animal — as well as the California condor, grizzly bear, northern gray wolf, and more. Today it protects more than 1,600 plant and animal species and 99% of the species placed on the endangered list have not gone extinct, says Jeremy Bruskotter, a professor at the School of Environment and Natural Resources at Ohio State University. [1] Also, because existing plant conservation funding is inadequate to support research, protection, and management of imperiled plants, a group of conservation specialists representing seven institutions in Florida began conversations in 2015 to initiate the Florida Rare Plant Conservation Endowment (FRPCE). The FRPCE is being established as a mechanism to provide long-term and reliable funding to support conservation-related projects for Florida imperiled plant species and their ecosystems. [2] Even so, the fund raised annually is far below the capital required to run all the protection projects of endangered plants.

1.2. Question Restatement

The general purpose of our work was to construct a model to give FRPCE Board an effective and reliable fundraising plan. Due to considerations of complexity of this decision-making problem, the scope of analysis was limited to only 48 imperiled species.

- First, we should focus on the relevant objectives in our fundraising model. In the following work, we then should try to find a method to evaluate whether a proposed fundraising plan is efficient and reliable on the basis of these objectives.
- Second, we should list and explain some general features of the imperiled species. We then need to determine the factors involved in the imperiled species' protection in fundraising decision model.
- Third, we need to develop a model that optimizes the arrangement of projects and output the solution as a timetable and a fundraising schedule.
- Last, we need to write a one-page non-technical memo, explain our results, and then give our recommendations on our model and analysis.

1.3. Our Work

We analyzed the characteristics of imperiled plants and the objectives of FRPCE, and incorporated and quantified them into the "integrated cost effectiveness." Then we built an optimization model, which also took the variation of feasibility of success with time (Model of Feasibility) into consideration. Afterwards, the model was solved by Monte Carlo Method. Finally, the model outputted the schedule and priority order of projects, and a fundraising schedule as well. The process is shown in the flowchart in Fig. 1.

Example 6: Problem B Introduction
Team 10876, Shanghai Pinghe School, Shanghai, China

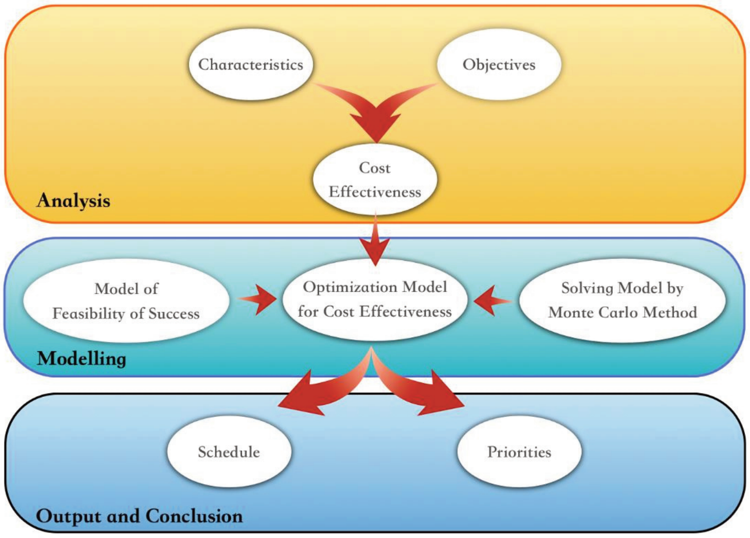


Fig. 1 Framework of This Paper

We also varied the inputs of the model and produced results under different circumstances: a higher budget limit or a linearly increasing budget limit. We provided further suggestions based on the results.

2. Assumptions

2.1. Assumptions and Justifications

- **Assumption 1:** FRPCE has a maximum capacity of fund raising each year. Also, FRPCE raises as much as the conservation projects that year cost, i.e., it cannot save money.
  Justification 1: FRPCE cannot make money by itself, and it relies on government and public donations. Its campaigns are unable to Donors and government officers would demand a financial report from FRPCE that demonstrates the use of the capital. The fund-raising must be completely need-based, and no excess capital is available to be passed on to the next year.

- **Assumption 2:** The maximum capacity of fund raising of FRPCE does not allow all the projects to start immediately. Some must be postponed, until more funding arrives in the future.
  Justification 2: If all the projects start simultaneously, the cost is more than ten times the money FRPCE put into plant conservation in the past year. [3] It is, apparently, unrealistic.

- **Assumption 3:** When a conservation project is postponed, the total cost and the benefit of successful conservation do not change, but the feasibility of success decreases. The rate of decrease is the same among all species.
  Justification 3: The total cost does not change because the approach of the project remains roughly the same. Also, in a relatively short time span (about 30 years,) the extent of uniqueness and benefit of the existence of a species is almost unchanging. The feasibility of success decreases because the size of population of the species decreases constantly until it becomes extinct. A smaller population means less offspring and less generic variation to counter environmental challenges.

3. Question Analysis

We analyzed the characteristics of the plants and the objectives of FRPCE and aimed at an optimization model incorporating them.

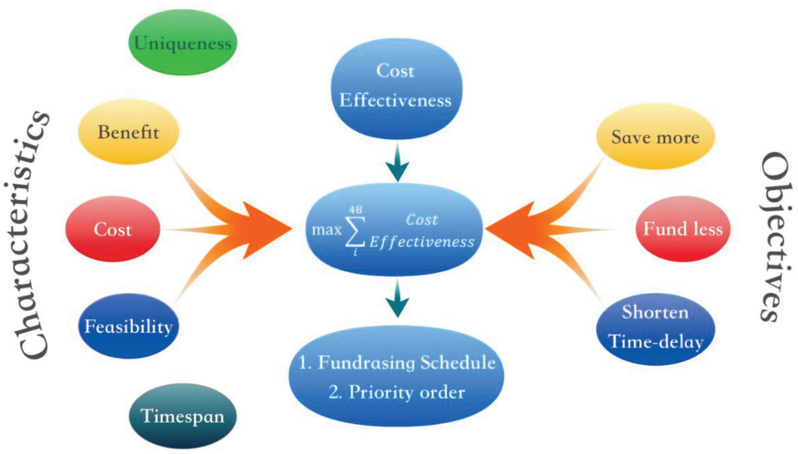


Fig. 2 Incorporating Characteristics of Plants and Objectives of FRPCE

3.1. Characteristics of Imperiled Plants

(For question 1b)
There are several characteristics that are indicative when deciding which species to recover:

- **Benefit of saving the plant:** This index implies relatively how beneficial saving this plant is. Plants with higher benefit are prioritized in conservation.
- **Taxonomic uniqueness of the plant:** The more unique a plant is, the more it contributes to biodiversity. Plants with high taxonomic uniqueness should be prioritized.
- **Feasibility of Success of Conservation:** Imperiled plants are often in harsh situations. Their conservation and recovery are often challenging, and success cannot be guaranteed. If a project fails after being invested, the funding is all wasted. Projects with high feasibility of success are prioritized.
- **The timespan of protecting a species:** Longer timespan leads to higher uncertainties of capital chain break, which cause the project to fail, and usually more funding is required by these projects. Conservation projects with shorter timespan are favored.
- **The total cost of a protection project:** Imperiled plants are delicate, and the protection of them is intricate, sophisticated and therefore, expensive. In contrast, the institutions that carry out the conservation projects often relies heavily on donations, and their capital is limited. Therefore, the less costly projects should be prioritized.

3.2. Objectives of FRPCE

(For question 1a)

We believe that the relevant objectives FRPCE Board need to consider in species protection and budgeting decision are the following.

- To save more species.

- To lower the amount of annual fund raised to lighten the fundraising burden.
- To shorten the time delay for conservation projects as much as possible: since the longer the time delay for a given project, the smaller the size of population and the lower the chance for that targeted species to survive.

However, these objectives above are mutually contradictory. For instance, limiting the budget each year indicates that there are fewer available projects, less benefits obtained, more project being postponed, and lower the chance of success of protecting species from extinction.

Thereout, we define a new index: integrated cost effectiveness, which evaluates the expected benefits obtained per unit capital per project and integrates benefits, feasibility of success and the cost for a single project together. Aiming at integrated cost effectiveness, we can plan a better fundraising schedule. This index will be elaborated into more details in section 4.2.

3.3. Fundraising Schedule and Priority Order

In order to determine the priority order of funding for the recovery projects and the corresponding fundraising schedule, the data given need to be closely examined. We especially focused on the Uniqueness, benefit and the annual cost of each recovery project.

- The upper-left part of Fig. 3 shows that there are only 3 types of Uniqueness: 1, 0.67 and 0.33. The major plants have uniqueness of 0.67. Only 2 are the most unique and 3 are least unique. We could first recover the most unique ones and then others later when solving the model.
- The benefit of projects consists of 4 different values. More than half of the projects have benefit of 0.66. We protect the plants with higher benefit first because we want to have maximized benefits, and postponing will lead to decrease of overall benefit.
- Also, “Flowering Plant-415” costs substantially more than the others as represented as the highest yellow line in right part of Fig. 3. Other than the outlier, the figure shows a trend of decline in cost for each project as time passes. So, we can imply that the cost needed will decrease once some projects started. Thus, putting off some projects to later years and wait until there is enough fund should be a workable method. However, the feasibility of a project will decrease if it is put off. In other words, the cost effectiveness gets lower when it starts late—this is not desired.

We will set up a model that maximizes the sum of the cost effectiveness of the conservation project of 48 plants. The model will be solved under the constraint of annual budget. The budget for every year and the recovery projects that are needed to be started in each year will be presented in the solution.

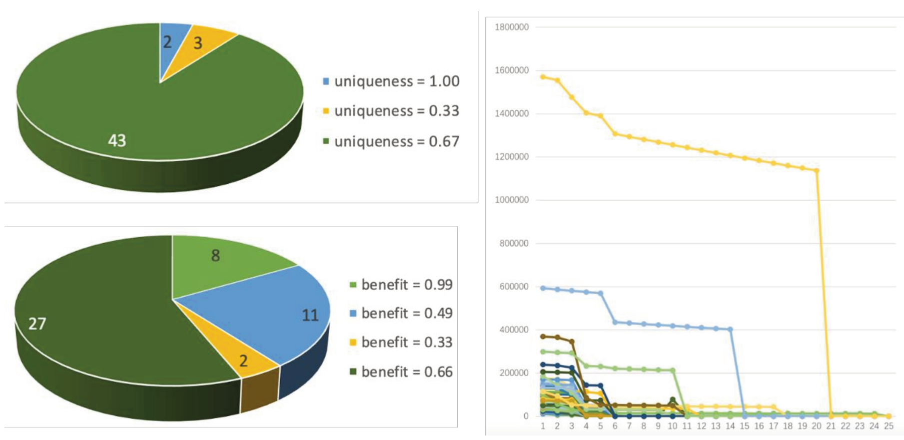


Fig. 3 Statistics of Taxonomic Uniqueness, Benefit and Cost

Example 7: Problem B Model Development, Continued
Team 10876, Shanghai Pinghe School, Shanghai, China

4. The Basic Model for Considered Variables (Questions 1 and 2)

The most basic model for evaluating the best summer job is a function between a summer job's utility and other independent variables, including the job's income, comfort, type of job, company environment, relevance to intended major/career, age and ability requirements, and risks. In order to evaluate the significance of each, we adopt the Analytic Hierarchy Process (AHP) to derive the weight for them. In order to analyze our model, we first outline and explain the variables we use for the study, then we explain the AHP model, and finally apply it on fictional characters for testing.

4.1 Deterministic Variables

Certain variables in jobs predetermine whether high school students may participate in the job. These are the basic requirements for the job, and they must be met as prerequisites. Because we are primarily concerned with high school students, one important consideration is the age requirement for the job. If a student is too young to undertake some cumbersome physical activities, they cannot choose the job. Another possible deterministic variable is ability prerequisite. However, our team decides to put that as a probabilistic variable, since in our opinion, in the age of high school, students should do something that can challenge themselves, which a skill-required job is more valuable than jobs with no ability requirement.

4.2 Probabilistic Variables

In this section, we consider variables that do not predetermine the types of jobs that students can choose. They are among the most important variables for high school students based on secondary sources [7] and in our perspective as high school students.

- **Income**

For many high school students, the income of a job is a crucial factor. Students who confront financial difficulties at home especially need jobs that offer higher salaries. The total income earned through working at a job, M, can be defined as

$$M = R \times T$$

where R is the hourly rate (in dollars per hour) and T is the total work time (in hours). We collect data on our summer job options' income using a variety of sources online, including Investopedia, Indeed, and Zipecruiter. Note that work time could also be a deterministic variable, as there is an upper limit for the amount of time available for students.

- **Comfort and Type of Job**

Especially during COVID-19, the nature of the job (i.e. virtual or in-person) could be a critical factor for selecting a summer job. We consider three independent probabilistic variables under this category, including virtual or not, sedentary or not, and the location of the job. The first variable depends on the local pandemic situation as well as individual preference for safety. The other two variables may depend on the student's personality and economic needs.

- **Company characteristics**

Company's features are factors usually considered by high school students as well. One consideration is the size of the company, which heavily influences student's decisions,

Example 8: Problem A Model Development
Team 10549, Basis International School Guangzhou, Guangdong, China

as some students may prefer individual work over teamwork. We vaguely define the size of the company, N, as the number of faculty employed.

- **Risks**

High school students likely want to minimize the danger associated with any summer job. One consideration may be the prevalence of COVID-19 in the region. According to Yonatan Grad, an epidemiologist at Harvard T.H. Chan School of Public Health,"The total incidence of [coronavirus infection] through 2025 will depend crucially on this duration of immunity"[8] Since this novel disease is unpredictable, we must take this into account as we are aiming at identifying the best summer job for students in 2021. As risks are often poorly defined, we adopt a Risk Score, RS, on a scale of 1-5 to assess the possibility for employees to encounter danger. One on the scale represents the lowest level of risks, which corresponds to jobs that are sedentary, indoor, or virtual. By contrast, works that require outdoor experience and first-aid experience are more likely to attain scale three. For all jobs, we would like to minimize RS. Risks of COVID-19 are also considered.

- **Skills**

High school students also intend to gain skills and experiences through summer jobs. Consequently, they want summer jobs that are relevant to their intended major or career. We define a skill index on a scale of 1-5 to determine how challenging the job is in terms of its ability requirement. For students looking for more advanced experience, jobs with higher ability index will fit them.

More compactly, the following is a schematic of the variables this study considers.

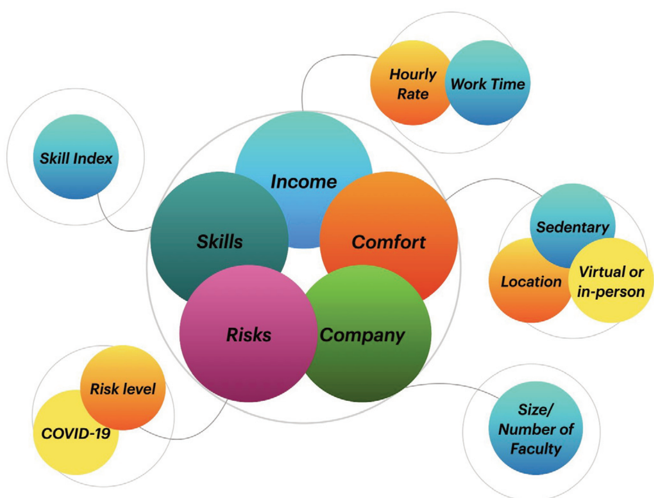
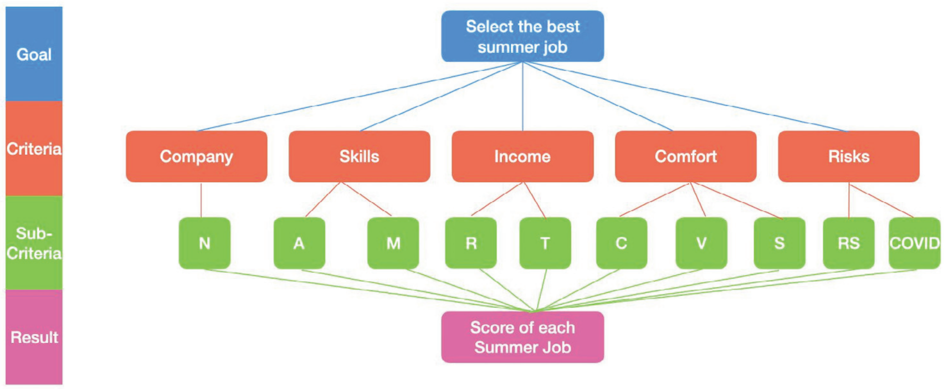


Figure 2: An overview of the variables this study considers

4.3 AHP Model

Having established the variables we need to consider, the following subsection will attempt to solve for the weights of each variable. This allows us to assess the relative importance of each variable, thereby letting us recommend the best summer job for each student user. In order to achieve this, the AHP model is an appropriate method. First developed by Thomas Saaty in the 1970s, the AHP model deals with a hierarchy of variables and can calculate the weights of each variable. A hierarchy structure of our variables is summarized below.

Figure 3: Hierarchy structure of variables



Variables Definition

Symbol	Definition
R	Hourly Rate of the summer job
T	Work time of the summer job
C1	Location of the company
V	Work completed virtually or in person
S	Work completed sedentary or not
N	Number of faculty/colleagues of the company
M	Relevance of the job to the student's future major
A	Ability requirement of the summer job
RS	Risk scale of the summer job
C2	Possibility of infecting COVID-19

AHP model effectively structures variables into a hierarchy. It is a way of assigning variables different weights based on the accompanied 1-9 scale table, which compares the importance of different factors. It determines the relative significance of each variable.

1-9 Scale Table

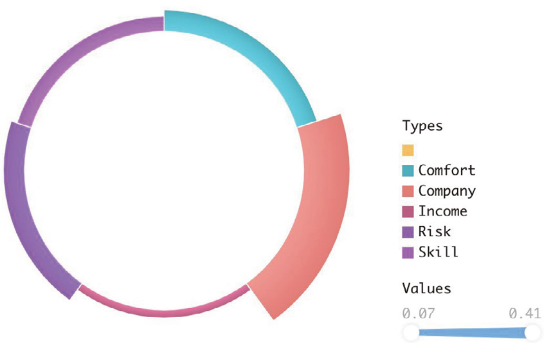
Scale	Definition
1	factor i is equally important as factor j
3	factor i is slightly more important than factor j
5	factor i is apparently more important than factor j
7	factor i is strongly more important than factor j
9	factor i is extremely more important than factor j
2, 4, 6, 8	the intermediate values
Reciprocal	importance scale between factor j and factor i

Based on the 1-9 scale table, it is possible to establish a comparison matrix for the variables. A comparison matrix is a square matrix of the following form. For each entry, C_{ij} represents the importance of C_i relative to C_j, as measured by the above scale table.

$$C=\begin{bmatrix} C & C_1 & C_2 & C_3 & \cdots & C_n \\ C_1 & C_{11} & C_{12} & C_{13} & \cdots & C_{1n} \\ C_2 & C_{21} & C_{22} & C_{23} & \cdots & C_{2n} \\ C_3 & C_{31} & C_{32} & C_{33} & \cdots & C_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ C_n & C_{n1} & C_{n2} & C_{n3} & \cdots & C_{nn} \end{bmatrix} \tag{3.1}$$

To evaluate the efficiency of the model, we construct a comparison matrix for a user whose primary concerns are comfort and skills gained rather than money. For this hypothetical example, we consider a high school student, Christopher, who values company and more than the income. The student's primary goal for the job is to expand his experience working in a large company. We accordingly compute her weights for the main criteria, as shown below:

Figure 4: Rose diagram of weights for criteria



3 The Stochastic Model

3.1 Objectives and Summary of the Stochastic Model

With the complexity of the issue, the S. Model focuses on the following primary objectives:

1. Fund all 48 projects
2. Minimize the total cost of all projects
3. Minimize the fluctuation of yearly cost
4. Reaching a target benefit value

The fundamental concept of the S. Model is to filter funding plans based on their estimated overall benefits and rank them according to their the estimated total cost. In the first stage of filtering, the overall benefit is divided into different subsets of factors: uniqueness, threaten level, ecological benefits, and feasibility of success. After the overall benefit of a funding plan is calculated, users need to provide a percentile value of the expected benefit with respect to the maximum benefit of all possible plans. Funding plans whose benefit is lower than this percentile are eliminated. Then, the remaining plans undergo another process of evaluation regarding the total cost of the plan and the deviation of yearly cost. This is achieved by multiplying the standard deviation of the yearly cost and an adjusted total cost to fund all project with respect to feasibility of success. The money required to fund a project is calculated by dividing the total cost of the project by its feasibility of success. It is higher if a project starts early in the process of the funding plan.

To fulfill the objectives, the S. Model takes into account all 48 projects in its random generation process of possible plans. The solution to the other three objectives are achieved by the characteristics of the algorithm. The specifics of algorithms are discussed in section 3.2 and section 3.3.

3.2 The Benefit of a Project with Respect to Time

The overall benefit of a project is determined by four major factors: The taxonomic uniqueness of a species, the extent to which a species is endangered, the ecological benefit yielded if the project succeeds, and the feasibility of success of a project. These factors are subjected to change over time. To address this issue, an algorithm is provided to evaluate the extent to which the change in these factors can affect the benefit outcome of the plan.

3.2.1 Taxonomic Uniqueness

According to section 1.2, the value $u_{i,0}$ is obtained from the "Taxonomic Uniqueness" column of the data spreadsheet. To enable further adjustment to the model, an exponential decay is used to express the relationship of the taxonomic uniqueness of a species to the starting time of its conservation project:

$$u_i = u_{i,0}\,(1 + r_u)^{s_i - 1}$$

In the equation, r_u denotes for a annual increment of taxonomic uniqueness over time.

In the HiMCM scenario, the values are scaled to [0,0.05] to adjust for the model. The default setting of the algorithm uses $r_u = 0.01$. Users can adjust this value to match the extent of change in the real world scenarios.

3.2.2 Threatened Level

The threatened level of a species reflects the state of which a species is endangered. (Section 1.2). In the HiMCM scenario, the initial value $T_{i,0}$ is obtained from the "benefit' column to indicate the Threatened Level. The Threatened Level undergoes an exponential increase if no measure is applied to conserve the endangered species (Section 2.2 Assumption 2):

$$T_{i,t} = T_{i,0}\,(1 + r_t)^{t-1}$$

Where $T_{i,0}$ is directly obtained from the "benefit" column (Section 2.2 Assumption 1). r_t is a weighting coefficient that determine the increasing Threatened Level over time. It is obtained from a region [0,1], in which 0 indicates no change and 1 indicates the most amount of change. It is theoretically possible to estimate r_t if a species' population data in previous years can be obtained. However, the weight r_t is a parameter that reflects the user's understanding of the imperiled species. In the HiMCM scenario, r_t is scaled to [0,1.25] with a default setting of $r_t = 1$.

3.2.3 Feasibility of Success

According to the assumption (Section 2.2 Assumption 6), the feasibility of success declines as the threatened level of a species increases. The actual feasibility of success with respect to the starting year of the project is calculated below:

$$f_i = f_{i,0} \cdot \frac{T_{i,0}}{T_{i,s_i}} = f_{i,0} \cdot \frac{T_{i,0}(1+r_t)^0}{T_{i,0}(1+r_t)^{s_i}} = f_{i,0}(1+r_t)^{-s_i}$$

The ratio between the initial threatened level and the threatened level when the project starts reflects the extent to which the feasibility of success will decrease over time. This is because if a species is more endangered, it is more unlikely for the project to succeed.

3.2.4 Ecological Benefits

To calculate the actual ecological benefit, one needs a more abundant data set that includes factors mentioned in 1.2. Due to the lack of information, the S. Model collects values of $x_{i,0}$ directly from the "benefit" column to represent the ecological benefit of a project. Since the ecological benefit of a project does not change over time, it is simply denoted as $x_{i,0}$.

3.2.5 Overall Score

The overall score of a project takes into account all the factors above. The formula of this calculation is given below:

$$bt_i = x_{i,0} \cdot f_i \cdot (1 - r_b u_i)^{s_i - 1}$$

Factor $x_{i,0}$ indicates that the higher the benefit of the project, the more need to initiate the project. Factor f_i indicates that a project with higher feasibility of success should be prioritized. Factor $(1 - r_b u_i)^{s_i-1}$ is more complicated, since the longer the project is postponed, the less benefit it will produce. Therefore, the benefit of the project undergoes an exponential decay over time.

In the calculation, r_u is a user-generated weighting coefficient that indicates the extent to which the benefit of a project decays over time. It is obtained from a region [0,5], in which 0 indicates no decay and 0.05 indicates the greatest decay. In the real calculation, r_u is scaled to adjust to the change of model outcome. In the HiMCM case, it is scaled to [0,0.05] with a default value 0.01.

To obtain the overall benefit of a plan, it is simply to take the sum of all bt_i;

$$bt = \sum_{i=1}^{48} bt_i$$

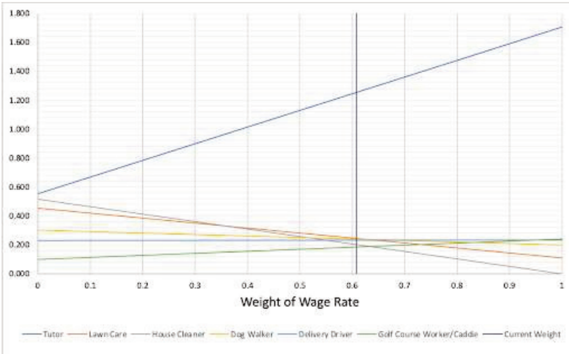
For the ith project, bt_i is maximized if the plan starts from year 1. Therefore, bt_i has a maximum if $s_i = 1$ for every $i \in [1, 48]$.

Example 9: Problem B Model Development, Continued
Team 10839, The McCallie School, TN, USA

5 Model Analysis

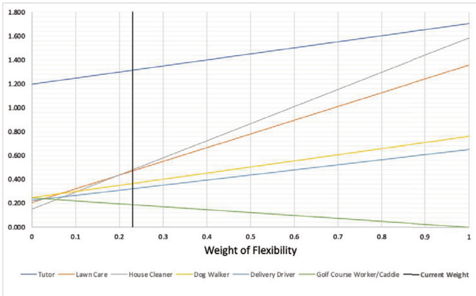
5.1 Sensitivity Analysis[4]

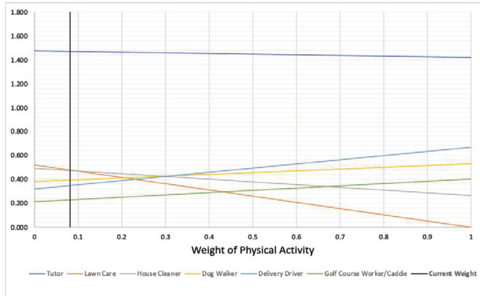
Our result shows that our model can make most suitable suggestions to the student based on the weights of their preferences. We also want to know how sensitive our model is, i.e., the stability of our job recommendation system under changes in the parameters. In particular, we want to know how changing weights of factors can affect final recommendation of jobs and by how much each factor can influence the rankings. To do this, we use our first model for Arthur and alter the weights of each factor while holding all other factors constant. From these results we can find the Absolute-Top (AT) factor, which is the factor that changes the top-ranked job with the smallest change in its weight. We decided to use AT as the most critical factor because we believe that students will pay more attention to the top-ranked job than any other job. Therefore, students will care about the AT the most compared to other factors.



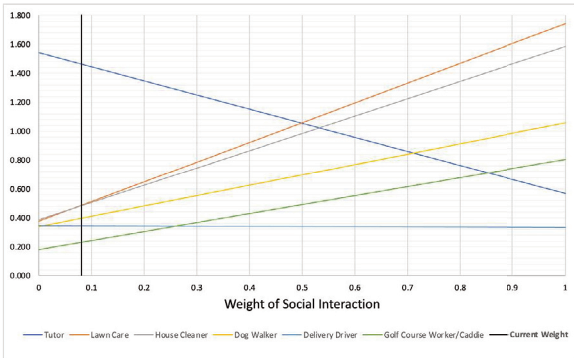
Notice the black vertical line indicates the weight produced from AHP, and its intersection with other lines indicates the final scores for those jobs. Moving the line left or right creates different points of intersections that create different final scores. Looking at the graph, for any weight of wage rate (holding the ratio of other weights constant), tutor will be the job recommended to Arthur because of his preferences. We repeat the same process for flexibility and physical activity and see a similar result:

[4]https://bpmsg.com/sensitivity-analysis-in-ahp/





Again, tutor is recommended to Arthur no matter the weights for the two factors holding the ratio of other weights constant. However, when we perform the analysis for social interaction, our result is more interesting:



At the current weight for social interaction, tutor surpasses all other jobs in the final score and is recommended to Arthur. However, if that weight increases to 0.498, then we see a change in the recommended job: it becomes lawn care. This change means social interaction is the Absolute-Top factor for Arthur, but it requires a substantial change in its weight to change the results.

5.2 Strengths

Our models is easy to implement and understand. It is well known that the Analytic Hierarchy Process can be easily implemented for any multiple criteria decision making. The conversion of qualitative variables and factors to quantitative measurements is reasonable and straightforward. Users can easily understand and indicate preferences because they only need to consider two factors. This is much simpler than trying to indicate preferences for all four factors at a time.

Our model provides a safeguard against inconsistent inputs. We calculate the Consistency Ratio (discussed in **3.1.1.1**) to make sure the student's preferences make sense and do not conflict. Otherwise, our system will make suggestions to the student that indicate which input(s) is/are relatively inconsistent. Other models cannot even provide protection from faulty data, less suggest ways to fix it. This makes our model very ***robust***.

Our models include a way to optimize preference of factors that is not used in traditional implementation of the Analytic Hierarchy Process. Conventional use of AHP requires the simple maximization or minimization of factors. Although there were factors in our model that needed to be maximized (wage and flexibility), we also had factors including social interaction and physical activity that needed to be tailored to the student's preference. We analyzed z-scores for this and were able to accurately calculate indices that were not just directly correlated with the levels of social interaction and physical activity. Our models take into account these preferences and are not as naïve as simply maximizing or minimizing factors.

5.3 Weaknesses

AHP has an arbitrary 1 to 9 scale that limits input. Firstly, the range is capped off at 9, so if a student believes factor A is 15 times more important than factor B, he/she has no way of inputting this extreme preference. Also, the scale makes it somewhat harder for a user to make a decision when comparing two factors that are extremely important to them.

AHP tends to take too long to collect user's response due to the massive amounts of comparisons required. With n factors, there needs to be $\binom{n}{2}$ comparisons between factors. Additionally, depending on its implementation, AHP sometimes also requires comparisons between alternatives (jobs in our models) regarding each factor. For our model, we were able to cut down the number of factors to make the number of comparisons feasible for the user. Additionally, we were able to cut out most of the comparisons between jobs by creating scales to rate each job (described in **3.2.2** and **3.3.2**).

6 Sensitivity Analysis

6.1 The influence of the cluster weights matrix

Cluster weights matrix comprises inter-cluster weights as well as weights (w1-w4 in Table 6) in relation to user's preferences. The cluster weights, w1-w4, are obtained by questionnaire so they are usually quite arbitrarily determined. We try to explore the sensitivity of cluster weights with an example of Student B (fictional person 2). Student B is athletic and has good physical fitness. The reason for looking for a summer job is to raise money, so in the cluster weights matrix, we set the weights that the Jobs cluster depends on others are 0.5 for salary (w1), 0.1 for benefit (w2), 0.2 for both work conditions (w3) and suitability (w4). Thus, there is no doubt that the best job for him/her is Athletic Fitness Model for Photo Shoot (Table 10), which offers a relative high salary and suitable for his physical condition. Now, let's assume that he/she has modified the goal. He/she wants to improve the leadership and teamwork skill rather than make money. The weights of salary and benefit should be switched (0.1 for salary and 0.5 for benefit). Then the job priorities (the 3rd column in Table 10) are totally different from the previous job priorities by pursuing high paid jobs. The best job for he/she is found to be Student Brand Ambassador and Summer Resident Camp Counselor is also a good choice. These two jobs meet the goal of improving leadership and teamwork skills. It's worth mentioning that with the adjusted pursuit the priorities of the jobs that are related to self-improvement have all increased.

The Cluster Weights matrix not only reflects the influence among clusters, but also exerts the influence of user preferences on the result normalization through W1-W4 (Table 5). W1-W4 is the most arbitrary weight in the whole super matrix, which is obtained by the user's survey questions. And we take Student B (Fictional person 2) best Job selection as an example, and discuss the influence of cluster weights on the decision results.

Table 10 Impacts of modified cluster weights on the job priorities for Students B and C

Jobs	Student B Original	Student B modified	Student C Original	Student C modified
Athletic Fitness Model for Photo Shoot	0.142	0.085	0.094	0.117
Data research Internship	0.072	0.088	0.084	0.082
Languages Teachers (online)	0.099	0.105	0.111	0.097
Library Technical Assistant	0.107	0.087	0.091	0.091
Part-time Copywriter	0.083	0.092	0.095	0.097
Student Brand Ambassador	0.093	0.129	0.118	0.106
Summer Resident Camp Counselor	0.078	0.120	0.120	0.104
Telephone Survey Advisor	0.104	0.105	0.095	0.103
Temp Retail	0.111	0.093	0.094	0.098
Warehouse Operatives	0.111	0.096	0.099	0.105

Besides, changes in between-cluster weights in the cluster weights matrix may also lead to different results. The 4th and 5th columns in Table 10 show the changes in resulting job priorities for the fictional person 8 (Student C) if the between-cluster weights are altered. This student has good physical fitness and hopes to improve leadership and gain some friendship, but lacks of professional skills. The best job for him/her is Summer Resident Camp Counselor, which meets his/her preference of improving leadership and gaining friendship. This result is under the condition that the influences of benefit, work conditions, suitability, and jobs clusters on the benefit cluster are 0.2, 0.1, 0.3, and 0.4, respectively (Table 6). If we assume that the influence of suitability on benefit is much greater than other clusters, which means the benefit we get from the job depends on whether we are suitable for the job. We set 0.1, 0.1, 0.6, and 0.2 as the weights to reflect this assumption. The resulting job priorities are changed as shown in the rightmost column of Table 10. The best job now turns to be Athletic Fitness Model for Photo Shoot, which is suitable for his/her good physical fitness. The jobs that may match his/her preference of improving leadership and gaining friendships are not the best choices because the student does not have professional skills given the new assumption that the suitability cluster mainly decides the benefit.

Example 11: Problem A Sensitivity Analysis, Continued
Team 10550, Nanjing Foreign Language School Xianlin Campus, Jiangsu, China

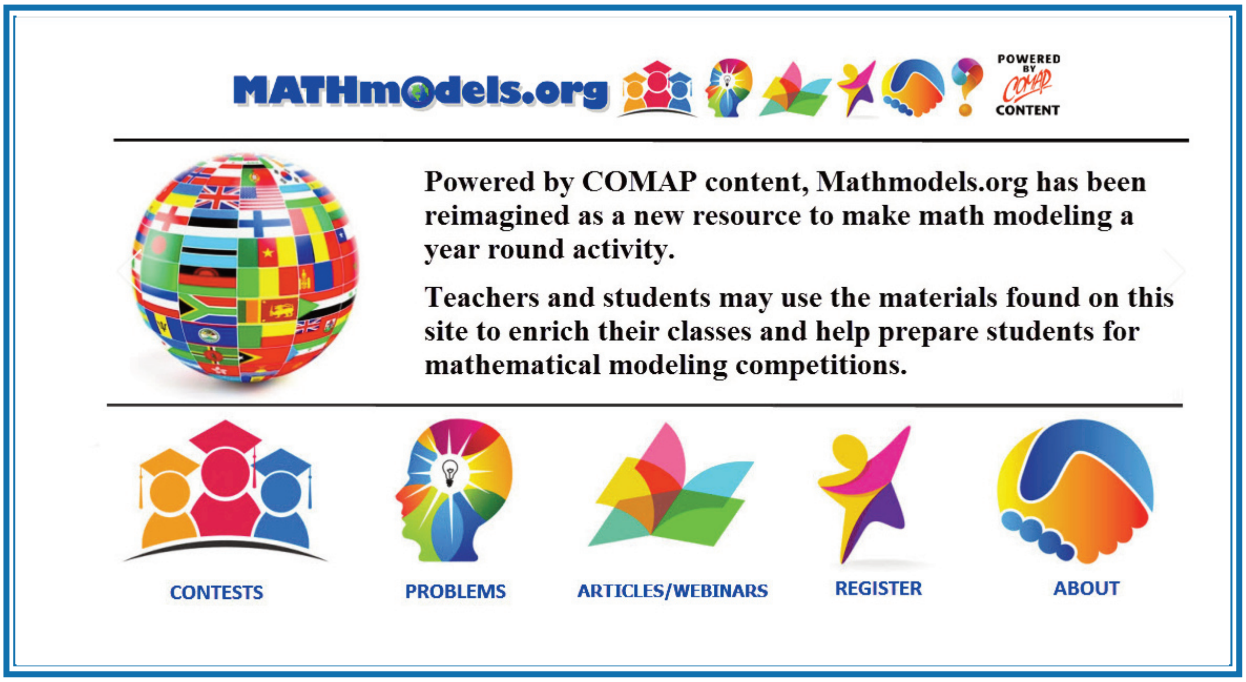
7. Conclusion

In this paper, we developed a model to find the best work opportunity for a high school student in the summer of 2021, whether that be a job or unpaid internship. This model is based off the user's input of a variety of relevant factors, such as COVID-19 risk, interests, and desired wage (in the case of a job). Each job and internship has assigned statistics for the same factors, and these are then combined to score each work opportunity with respect to the user. Two different models were created for jobs and unpaid internships, because they operate off a slightly different set of factors. However, the models are very similar in operation and sensitivity.

The model was tested with 17 different data inputs in the form of fictional students, with 10 students seeking jobs and 7 seeking internships. Jobs were selected for each of the 10 users from a database of 36 jobs, as curated in Section 3.2. A similar list of 25 internships was developed for the 7 users seeking those opportunities. The input data was designed to be as realistic as possible while covering a large gamut of possible inputs to the model. The model was found to work sufficiently in all input cases, providing reasonable work opportunities for all users, as desired. These opportunities matched the user's preferences as well as possible from the small database available. We posit that, with a larger database of potential work opportunities, matches will only become more accurate.

The model was presented using a website for the greatest ease of entering user input as well as the fact that all calculations could be run automatically. In the future, we desire to fully implement this model. We also hope to improve the model by factoring in commute and favoring jobs that are closer to home over jobs that are farther away. While it was infeasible to manually add commute data for each job on each input case, this could be done in a real implementation via a web scraper that scrapes job data from online using the user's location. Another factor worth considering is qualifications. These were disregarded in the current model via Assumption 4, but the inclusion of these would make the model more accurate and realistic.

Example 12: Problem A Conclusion
Team 10656, Massachusetts Academy of Math and Sciences at WPI, MA, USA



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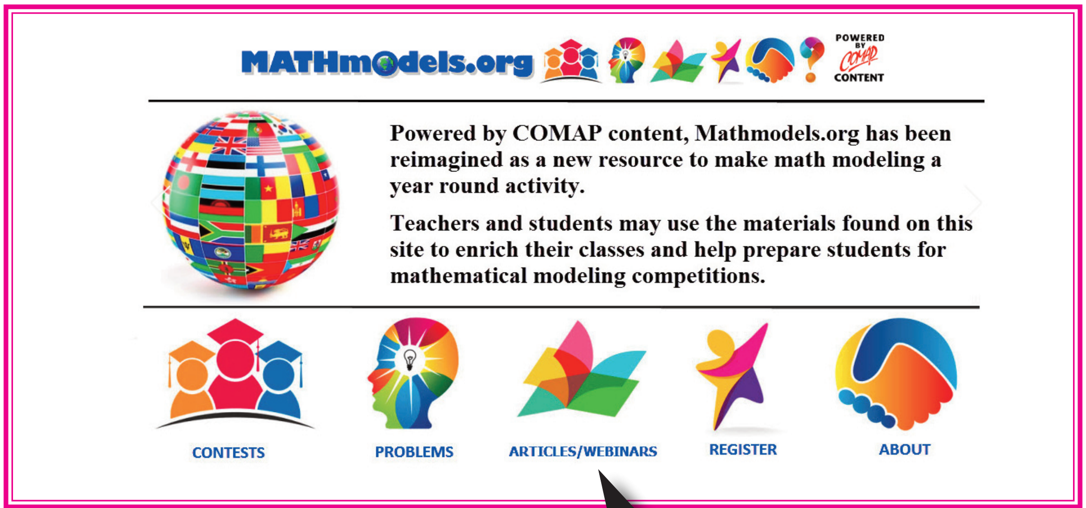

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