Increasing student engagement using Math modeling and drones in a mathematics classroom

Aziz Zahraoui

(Pronounced: Za’Ra’Wii -------------Twitter: @MrZchs)

Secondary Mathematics Teacher (Geometry/ AP Calculus)

Community College Adjunct Faculty (Quantitative Reasoning/ Pre-Calculus)

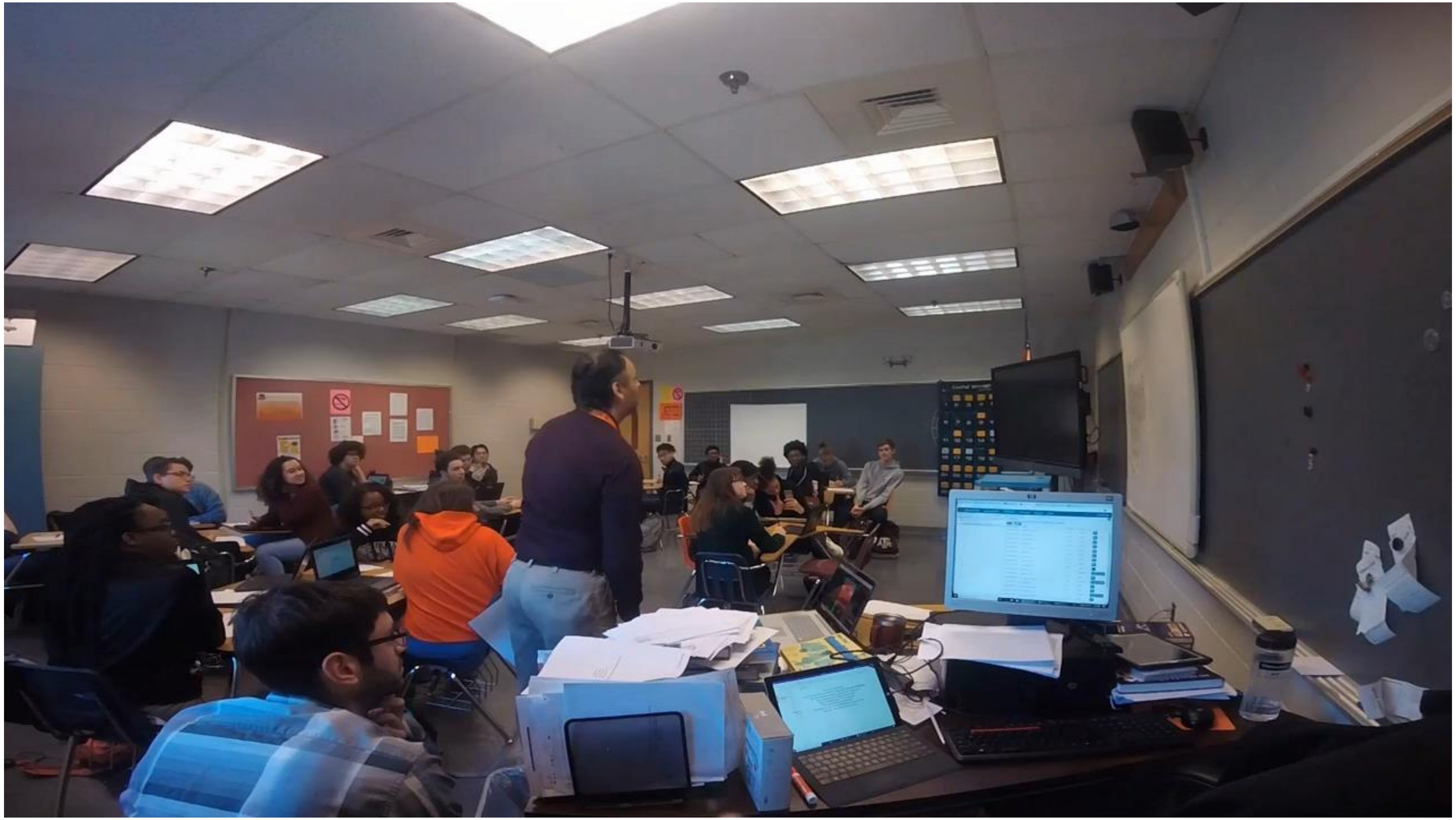
NASA MODSIM Teacher Ambassador and Math Modeling Hub Fellow

How it started

The Problem:

The Tello drone has 2 sensors, 1 sensor measures the acceleration of the drone, and the other sensor measures the height (altitude) of the drone. The drone provides data about 3 measurements, height, velocity, and acceleration. If the Tello drone does not have a sensor for velocity, how does it provide measurements for velocity? How can we verify that the sensors are functioning properly onboard the drone?

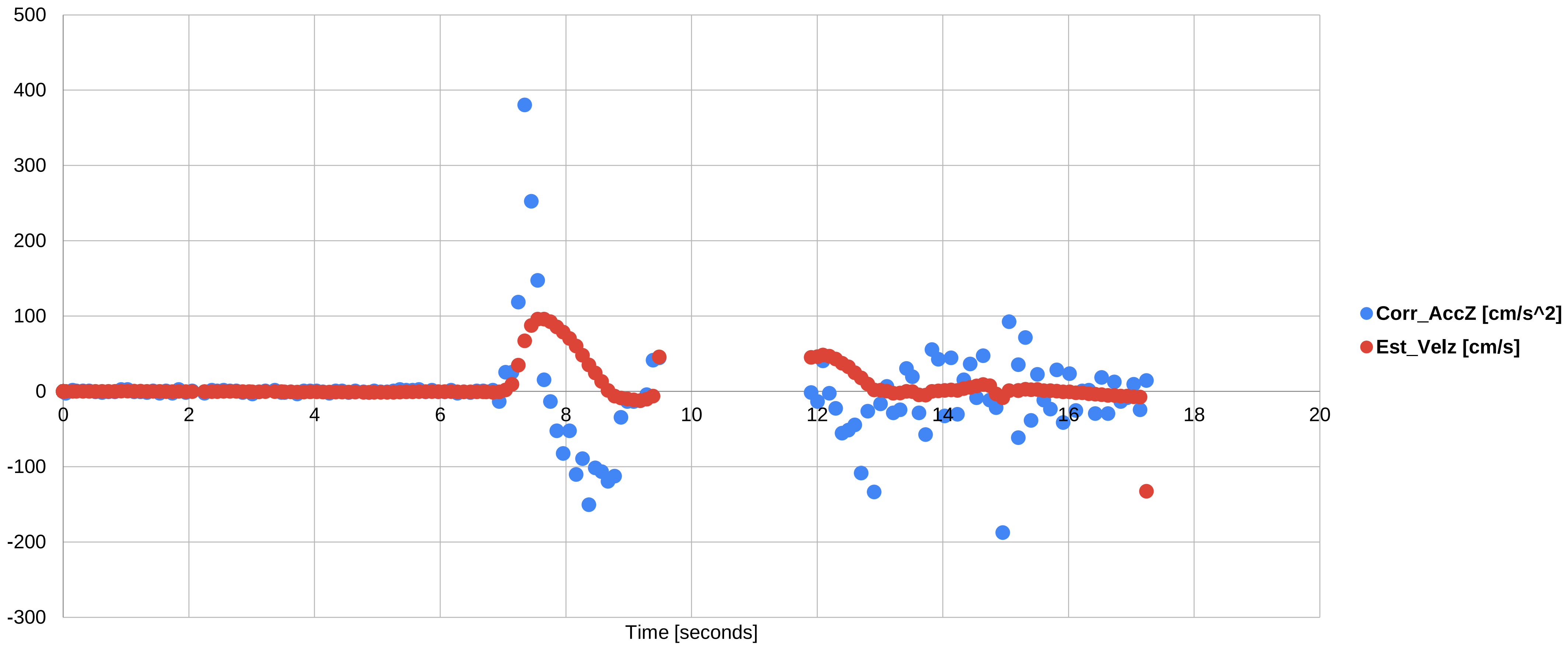
The Drone Flight in-Class 3/28/2019



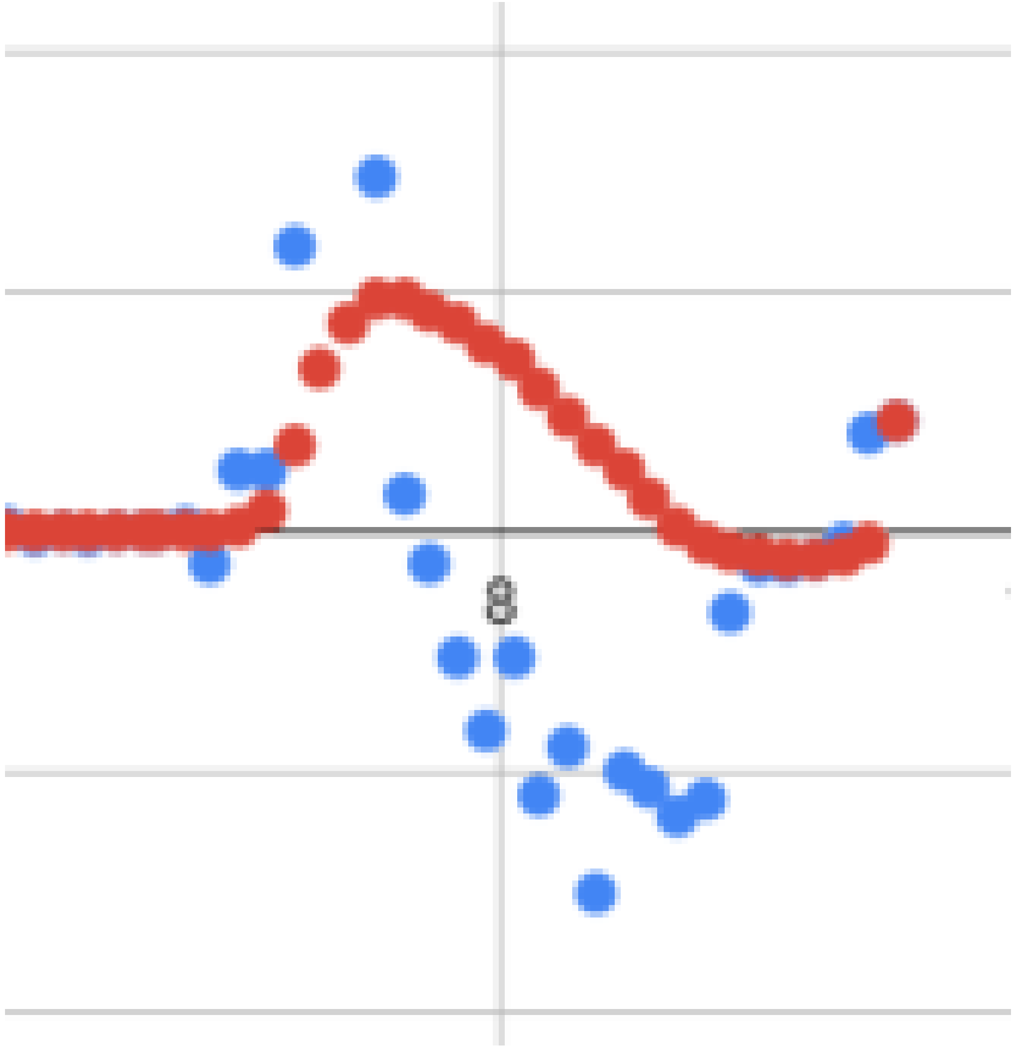
Data Collected

Data from 3/28/2019				Initial Conditions	0	0	
time [s]	Height [cm]	Velz [cm/s]	Accz [cm/s^2]	Corr_AccZ [cm/s^2]	Est_Velz [cm/s]	Est_Height [cm]	Pos from Vel
0.04	0	0	-993	-2.5	-0.0535	-0.0025145	0
0.147	0	0	-997	1.5	0.0065	-0.0005495	0
0.207	0	0	-996	0.5	0.059	0.00829675	0
0.312	0	0	-996	0.5	0.1095	0.01935625	0
0.413	0	0	-996	0.5	0.1095	0.02511325	0
0.514	0	0	-995	-0.5	0.0045	0.02028325	0
0.619	0	0	-994	-1.5	-0.0965	0.0079865	0
0.72	0	0	-995	-0.5	-0.147	-0.0017095	0
0.821	0	0	-995	-0.5	-0.045	0.006578	0
0.923	0	0	-998	2.5	0.2075	0.032737	0
1.024	0	0	-998	2.5	0.3105	0.062092	0
1.127	0	0	-995	-0.5	0.2595	0.083359	0
1.229	0	0	-995	-0.5	0.1575	0.0968995	0
1.331	0	0	-994	-1.5	0.108	0.102493	0
1.43	0	0	-996	0.5	0.005	0.0977035	0
1.533	0	0	-993	-2.5	-0.098	0.082305	0
1.636	0	0	-996	0.5	-0.201	0.061602	0
1.739	0	0	-993	-2.5	-0.201	0.044302	0
1.839	0	0	-998	2.5	-0.145	0.025262	0
1.951	0	0	-994	-1.5	-0.195	0.000862	0
2.051	0	0	-996	0.5	-0.293	-0.056958	0
2.247	0	0	-994	-1.5	-0.297	-0.057609	0
2.249	0	0	-993	-2.5	-0.354	-0.092949	0
2.363	0	0	-997	1.5	-0.266	-0.111825	0
2.451	0	0	-996	0.5	-0.163	-0.1234125	0

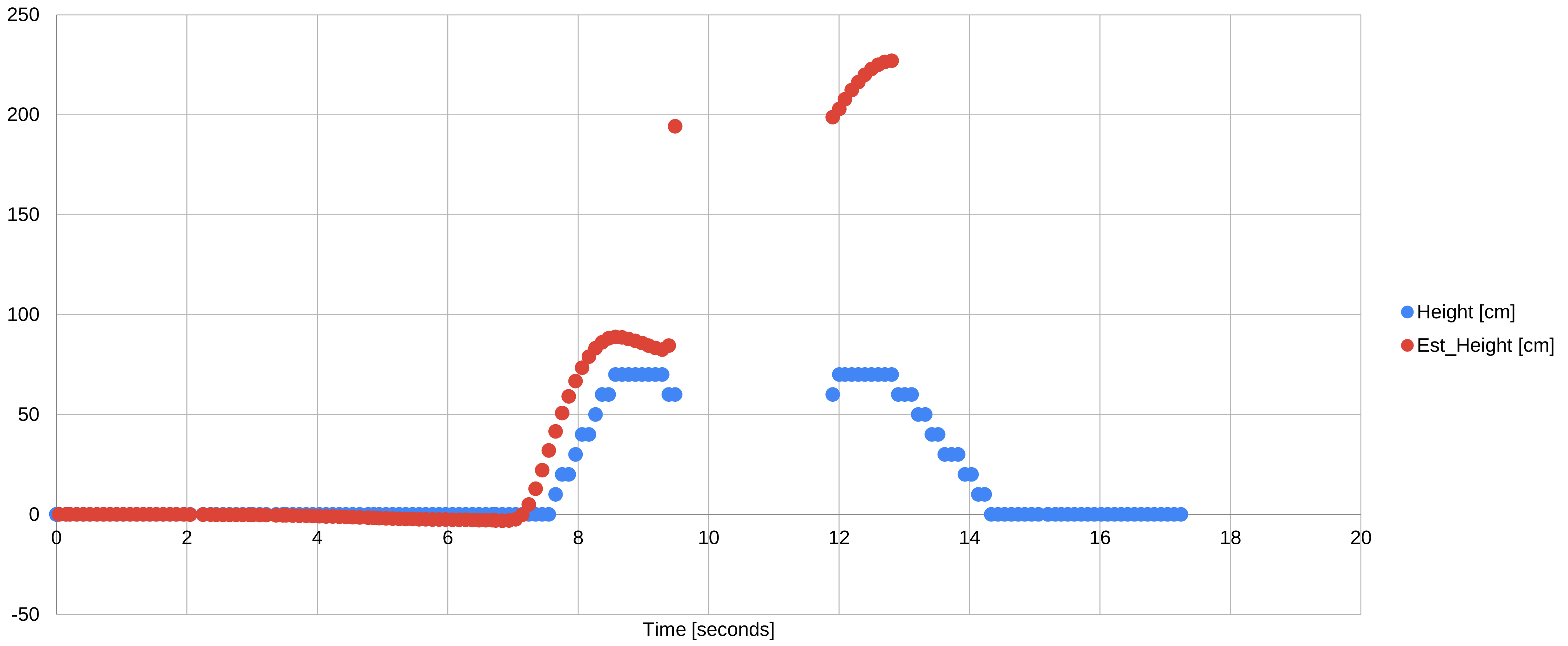
Acceleration and Velocity Graphs--Real



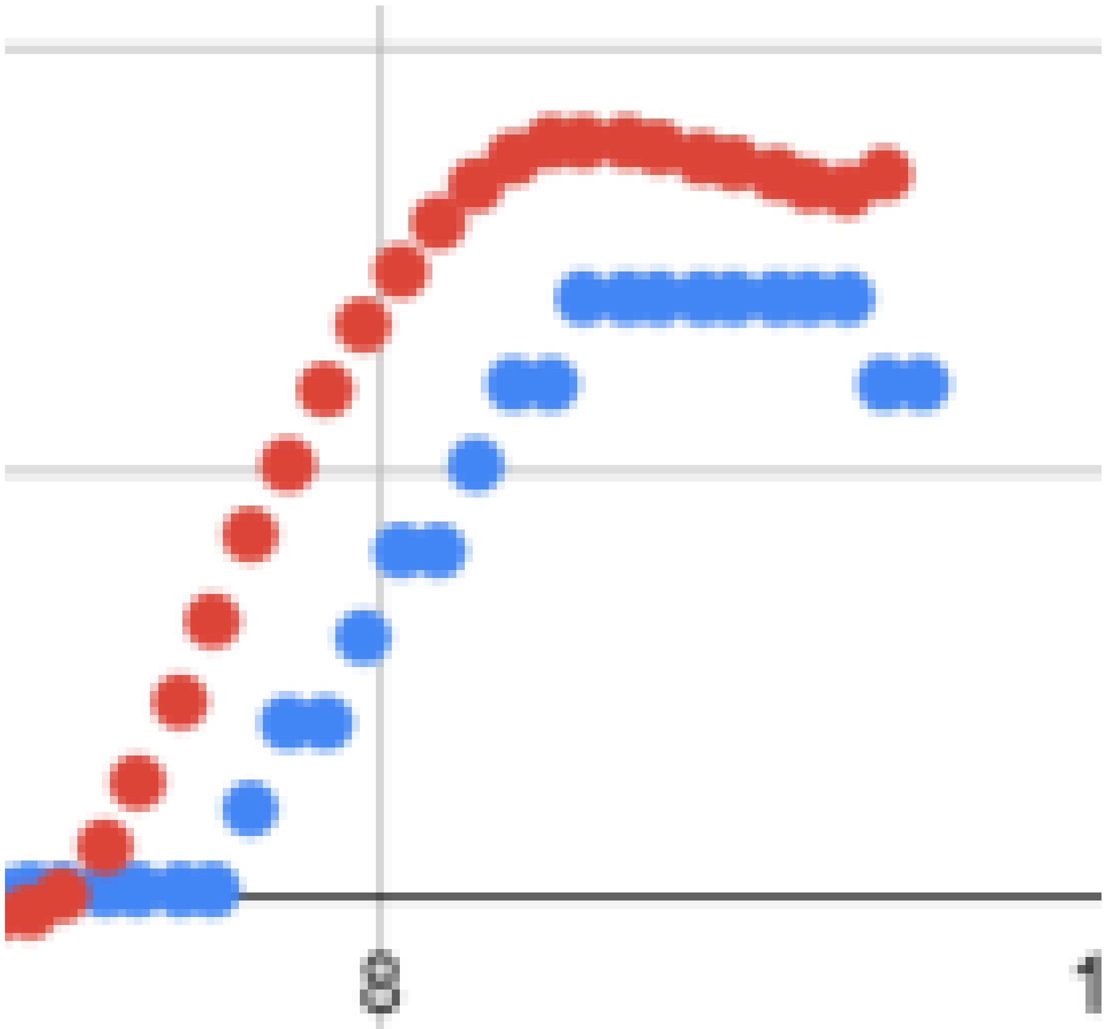
Acceleration and Velocity Graph



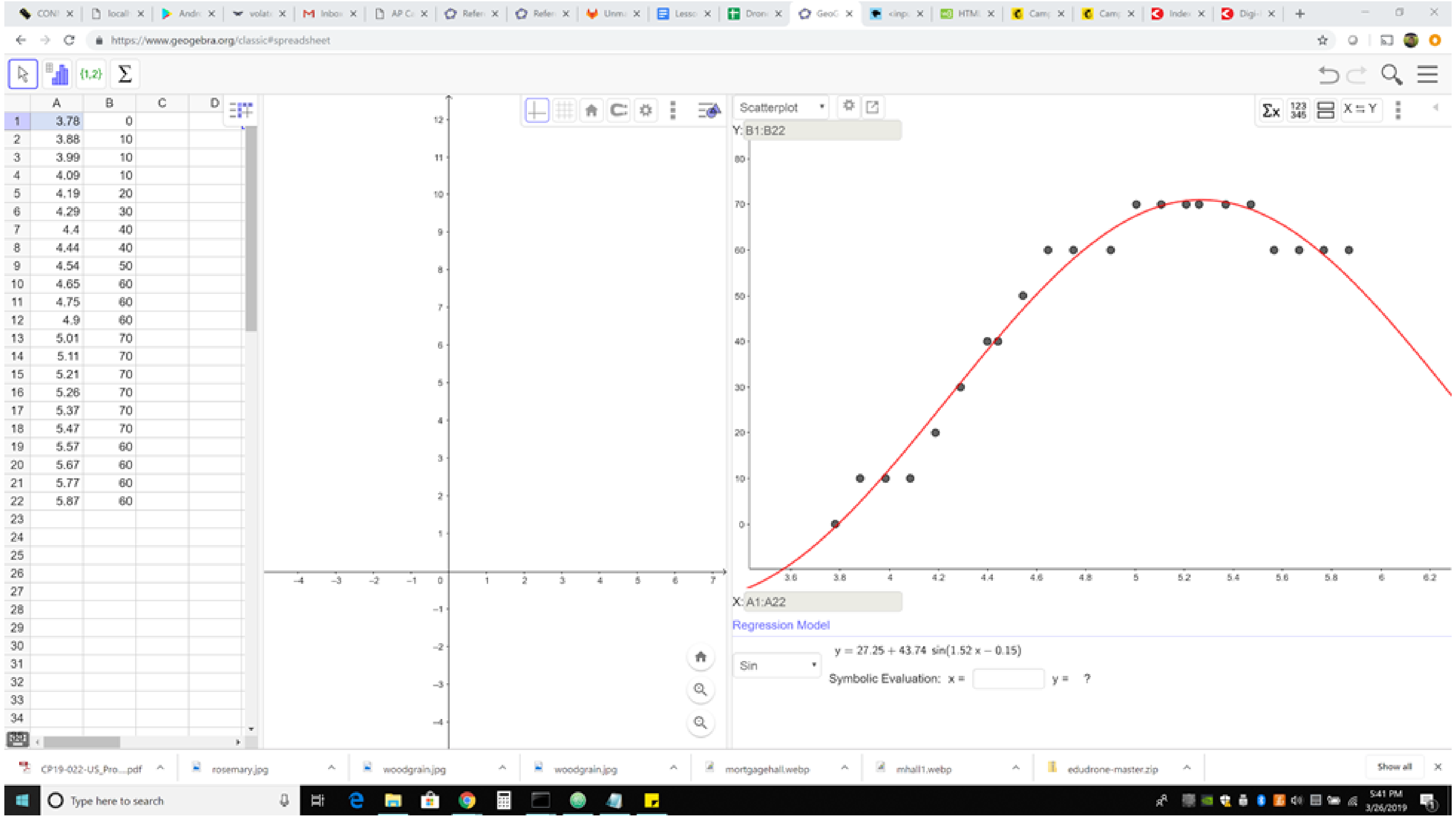
The height Graph-- Real



Measured Height compared to estimated height



Geogebra Spreadsheet of a segment of height data vs time. This is used to estimate a continuous function that is differentiable within the time segment.



Name:____________________ Group:________

What is a drone? It flies, it makes noise, but what is it?

You've seen the drone takeoff and land.

The instantaneous moment that the drone lifts off the table, what is the position, velocity, acceleration? Draw the scenario if it helps. Note that its initial state is position = 0, velocity = 0, acceleration = 0

What is the state of the drone when it is hovering in the air? Again, draw the situation if it helps.

Describe the acceleration and velocity for the period of time that it moves from its initial position to its hover position. Use a graph of acceleration vs time, and velocity vs time to present your argument.

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

t (seconds)	0	12	20	24	40
$v(t)$ (meters per second)	0	2	6	–4	8

3. Kyani, a drone pilot, flies her drone along a vertical path (GPS Column). For $0 \le t \le 40$, Kyani's drone velocity is given by a differentiable function v. Selected values from the flight data recorder of $v(t)$, are given in the table above. Where t is measured in seconds and $v(t)$ is measured in meters per second.

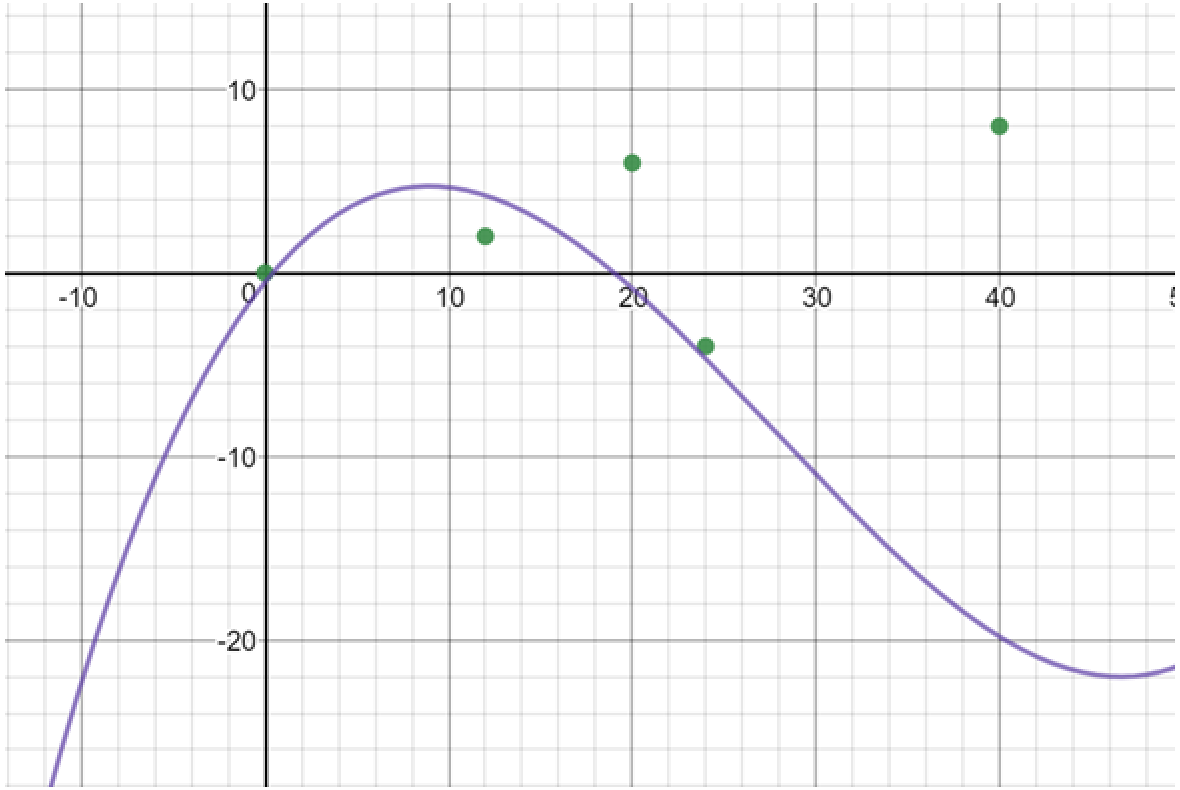
(a) Use the data in the table to estimate the value of $v'(21)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

(c) Malik is flying his drone along the same path. For $0 \le t \le 20$, Malik's velocity is modeled by $M(t) = \frac{t^3}{1000} - \frac{t^2}{12} + \frac{5}{4}t - \frac{1}{2}$, where t is measured in seconds and $M(t)$ is measured in meters per seconds. Find Malik's drone acceleration at time $t = 5$.

(d) Based on the model M from part (c), find Malik's average velocity during the interval $0 \le t \le 20$.

Geogebra Graph of the table of points presented in the Pre-Test and the continuous function generated as a model.



(a) $v'(21) = \frac{-4-6}{24-20} = -\frac{10}{4} = -2.5$ meters/sec^2

1: approximation

(b) $\int_0^{40} |v(t)|\, dt$ is the total vertical distance Kyani's drone flew, in meters, over the interval $0 \leq t \leq 40$

$$\begin{aligned} \int_0^{40} |v(t)|\, dt &\approx 12 \cdot |v(2)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 2 + 6 \cdot 8 + 4 \cdot 4 + 16 \cdot 8 \\ &= 216 \text{ meters} \end{aligned}$$

$3: \begin{cases} 1: explanation \\ 1: right\ Rieman\ sum \\ 1: approximation \end{cases}$

(c) Malik's Acceleration is $M'(t) = \frac{3}{1000}x^2 - \frac{x}{6} + \frac{5}{4}$

$M(5) = \frac{3}{1000}(25) - \frac{1}{6}(5) + \frac{5}{4} = \frac{59}{120} \approx 0.491$ meters/sec^2

$2: \begin{cases} 1: uses\ M'(t) \\ 1: answer \end{cases}$

(d) $Avg\ vel$

$$\begin{aligned} &= \frac{1}{20}\int_0^{20} \left(\frac{t^3}{1000} - \frac{t^2}{12} + \frac{5}{4}t - \frac{1}{2}\right) dt \\ &= \frac{1}{20}\left[\frac{t^4}{4000} - \frac{t^3}{36} + \frac{5}{8}t^2 - \frac{1}{2}t\right]_0^{20} \\ &= \frac{1}{20}\left[\frac{20^4}{4000} - \frac{20^3}{36} + \frac{5}{8} \cdot 20^2 - \frac{1}{2} \cdot 20\right] \\ &= \frac{26}{9} \approx 2.889 \text{ meters/sec} \end{aligned}$$

$3: \begin{cases} 1: integral \\ 1: antiderivative \\ 1: answer \end{cases}$

What worked?

- **Bringing the drone to class was a hit with students**, some asked to take short videos to post on social media. It was nice seeing the kids excited about learning in a math class.
- **Trying to solve a real-life engineering problem created a sense of purpose for students**. A sense that they are not doing the problem as a drill, but more to solve a well-defined engineering problem.
- **The conversations my students were having throughout the session were on target and constructive.** They did a good job activating prior knowledge and using it to solve the problem in front of them.

EK 3.3A3: Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x)=\int_a^x f(t)dt$.

Metric Data for the drone lesson

Learning Standard	Pre-lesson [# students]		Post-lesson [# students]	
Essential knowledge (EK)	Proficient	Sub standard	Proficient	Sub standard
EK 2.1B1: The derivative at a point can be estimated from information given in tables or graphs.	15	10	24	1
EK 3.2A1: A Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.	13	12	22	3
EK 3.3A3: Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x)=\int_a^x f(t)dt$.	10	15	19	6

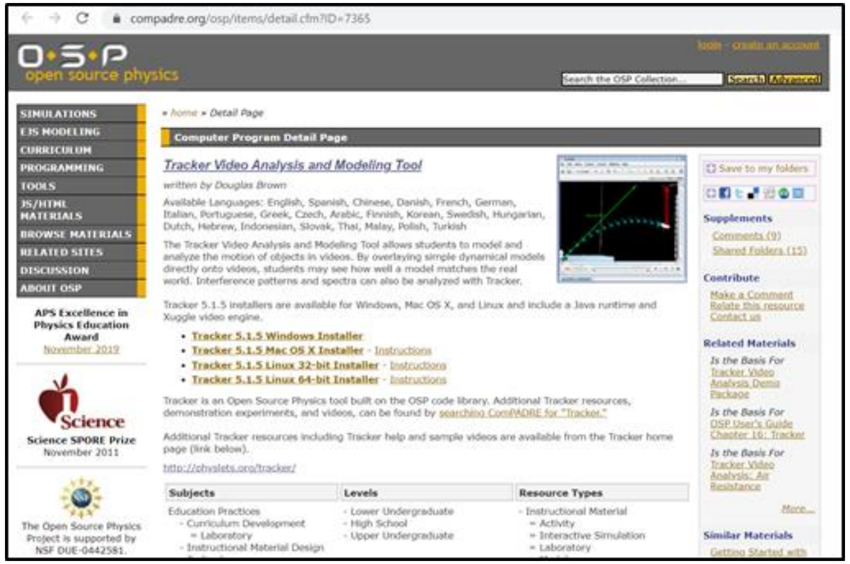
What did not work?

1) **The Student Companion Guide did not do a good job.** The questions were too broad, leaving me with the task of doing more to steer the students in the right direction. I needed to have used detailed questions to focus the students on the right aspects to consider. This is also where a teacher might consider modifying the activity to better fit their specific students.

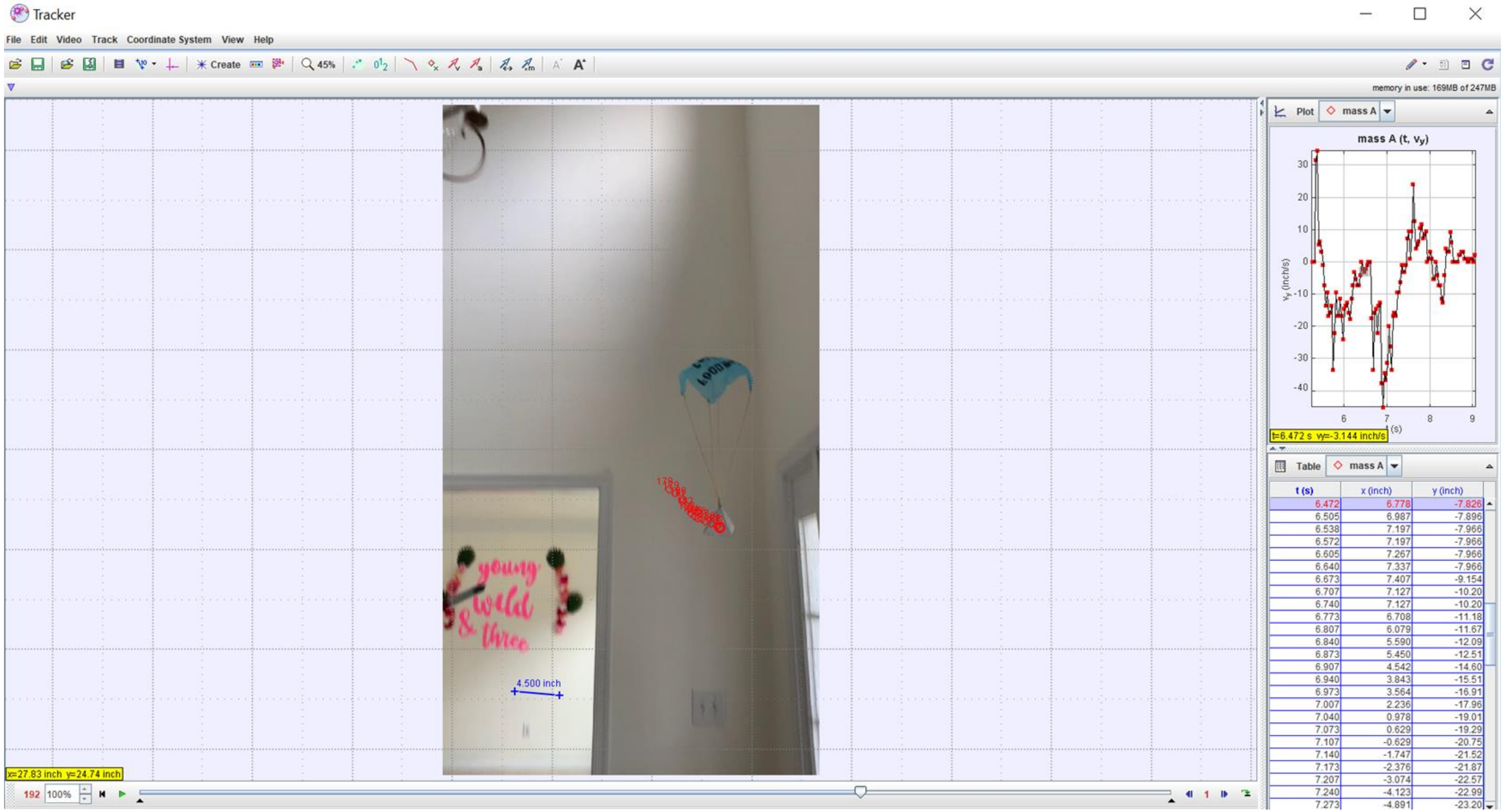
2) **Using google classroom to disseminate the data obtained from the drone was not done properly.** I sent the data ahead of time to eliminate transition downtime. Unfortunately, some students accessed it ahead of me. I rolled with it even though it was not planned that way.

3) **The graphs attached to the data did not show up the way I would have liked them to be**. They were small and hard to maneuver. They could still serve the purpose. A better idea would have been to take the data and have the students feed it to Geogebra, this would have extended the scope of the lesson beyond what I wanted. Nonetheless, moving forward, that is what I will do. I would also use this to circumvent any technical issues. I would fly the drone, and then use data from a similar flight that I have saved. Collecting data instantly can be a technical and pedagogical nightmare.

What to do if you do not have a drone?



The Tracker



Tracker
File Edit Video Track Coordinate System View Help
Create
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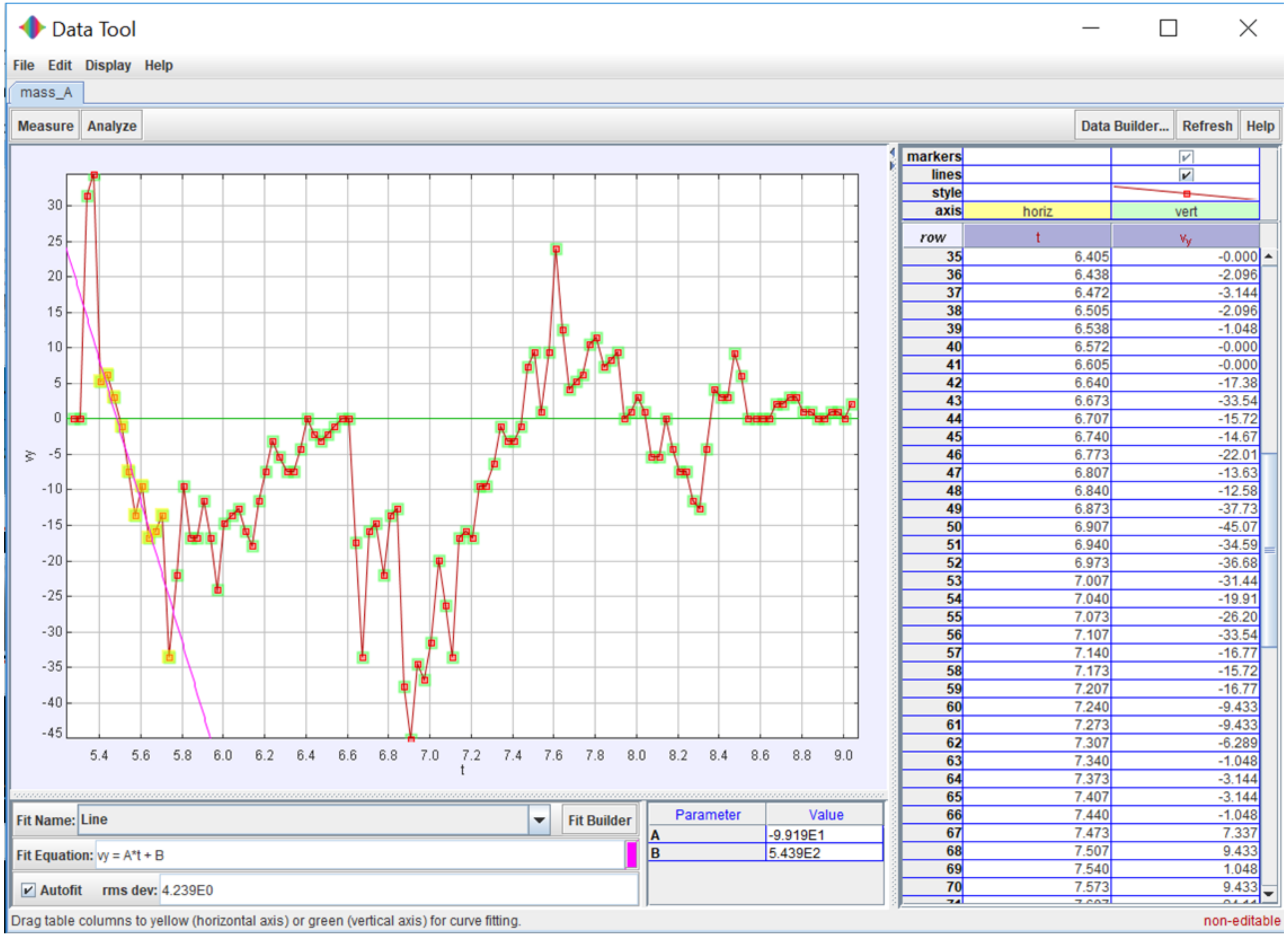
Plot
mass A
mass A (t, v_y)
v_y (inch/s)
t (s)
t=6.472 s vy=-3.144 inch/s
Table
t (s)
x (inch)
y (inch)
4.500 inch
x=27.83 inch y=24.74 inch
192
100%

The Tracker



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35 6.405 -0.000
36 6.438 -2.096
37 6.472 -3.144
38 6.505 -2.096
39 6.538 -1.048
40 6.572 -0.000
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42 6.640 -17.38
43 6.673 -33.54
44 6.707 -15.72
45 6.740 -14.67
46 6.773 -22.01
47 6.807 -13.63
48 6.840 -12.58
49 6.873 -37.73
50 6.907 -45.07
51 6.940 -34.59
52 6.973 -36.68
53 7.007 -31.44
54 7.040 -19.91
55 7.073 -26.20
56 7.107 -33.54
57 7.140 -16.77
58 7.173 -15.72
59 7.207 -16.77
60 7.240 -9.433
61 7.273 -9.433
62 7.307 -6.289
63 7.340 -1.048
64 7.373 -3.144
65 7.407 -3.144
66 7.440 -1.048
67 7.473 7.337
68 7.507 9.433
69 7.540 1.048
70 7.573 9.433
vy
t
Fit Name: Line
Fit Builder
Fit Equation: vy = A*t + B
Autofit rms dev: 4.239E0
Parameter Value
A -9.919E1
B 5.439E2
Drag table columns to yellow (horizontal axis) or green (vertical axis) for curve fitting.
non-editable

Questions?

Email: Aziz.zahraoui@portsk12.com

@MrZchs